Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

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в	г	е	ё	ж	и	й	у	х	ц	ч	ш	щ	ъ	ы	ь	э	ю	я
v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	″	ɨ	′	è	yu	ya

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September 1950 Vol. 3, No. 9

Applied Mechanics Reviews to Be Edited at Midwest Research Institute

During the initial period of development and growth of the Applied Mechanics Reviews, part of the financial burden has been shared by The American Society of Mechanical Engineers and the Illinois Institute of Technology. The Institute finds itself unable to continue the substantial contribution which the editors feel is essential if Applied Mechanics Reviews is to achieve the permanent place its sponsors have hoped to reach. Therefore, the Illinois Institute of Technology is withdrawing from active sponsorship, and the editorial offices are being transferred to the Midwest Research Institute at Kansas City, Missouri, which will produce copy for Applied Mechanics Reviews beginning with the October, 1950, issue.

Both previous editors affiliated with the Illinois Institute of Technology wish to thank our reviewers for their magnificent co-operation, and our readers for their sympathy and forbearance during the development period. The Editor

Communications

Correction

In Rev. 3, **1168**, the signature should be C. J. Bouwkamp, Holland (not "Germany"). Ed.

Theoretical and Experimental Methods

(See also Revs. 1636, 1649, 1676)

1616. **E. M. Wilson, A note on the numerical integration of differential equations,** Quart. J. Mech. appl. Math. 2, 208–211 (June 1949).

A method is presented which, though not of universal application, can often be applied to the numerical solution of linear differential equations. The method makes use of a Taylor-series expansion of the solution about a point, the various derivatives required to construct the Taylor series being obtained by successive differentiation of, and substitution in, the original differential equation. A tabular procedure for a point-by-point solution in some cases becomes relatively simple by this method. Treatment of the first-order linear differential equation is presented in detail with an illustrative example that demonstrates the high degree of accuracy that can be attained; the method of solution of higher-order equations being indicated by inference. The author points out that in some cases the calculations may be uneconomic unless the new types of automatic computing machines, capable of carrying out a continuous series of multiplications and additions, are used. S. S. Manson, USA

1617. **L. Fox and E. T. Goodwin, Some new methods for the numerical integration of ordinary differential equations,** Proc. Camb. phil. Soc. 45, 373–388 (July 1949).

Several numerical methods are discussed for a step-by-step integration of ordinary differential equations, including first- and second-order equations, and simultaneous sets of first-order equations. In contradistinction to previous methods, such as that of central differences, the methods of this paper involve little or no differentiation or estimation of higher-order differences, and dispense wholly or in part with the computation of initial values. The use of a difference correction technique obviates the necessity for an iterative process at each step of integration.

The formulas of this paper are expressed in the form of a recurrence relation plus a correction term Δ. For instance, to solve the first-order equation, $y' = fy + g$, method II of the paper uses the formula

$$y_1 = y_0 + \tfrac{1}{2}h\,(y_0' - y_1') + \Delta,$$

$$\Delta = \left(-\frac{1}{12}\,\delta^3 + \frac{1}{120}\,\delta^5 - \frac{1}{840}\,\delta^7 + \ldots\right) y_{1/2}.$$

The recurrence relationship is then with the correction term

$$(1 - \tfrac{1}{2}\,hf_1)y_1 = (1 + \tfrac{1}{2}\,hf_0)y_0 + \tfrac{1}{2}\,h(g_0 + g_1) + \Delta.$$

Some comparison is made of the efficacy of the seven methods presented in the paper. Stanley Thompson, USA

1618. **K. Steinbuch, A method of solution of one-dimensional diffusion phenomena** (in German), Ingen.-Arch. 17, no. 3, 233–242 (1949).

The author describes a method of successive approximations for the solution of certain boundary-value problems for the equation of diffusion $\partial v/\partial t = c\partial^2 v/\partial x^2$. The equation is supposed to be satisfied in an infinite strip $0 \leq x \leq x_1$, the system being at rest at time $t = 0$. The boundary conditions (displacement or velocity) are specified for $x = 0$, $t > 0$, while for $x = x_1$, $t > 0$, the boundary condition is determined by the fact that the shearing forces at the wall are supposed to move a rigid body, e.g., a flat lamina. It is well known that if the fluid is not bounded on the right ($x_1 \to \infty$), the solution can be expressed very simply in terms of the error function, and it is shown in the present paper how the more complicated problem considered here can be solved by an extension of the familiar method of images. A convenient symbolism is developed for carrying out the procedure, and it is shown that good convergence may be expected in practice. The method is applied numerically to the calculation of the transmission of motion across a liquid layer between two concentric rigid

cylinders. It appears to the reviewer that a modification of the procedure is desirable in this particular case, since the one-dimensional equation of diffusion does not, strictly speaking, apply to it. A. Robinson, USA

1619. Carl A. Ludeke, An electro-mechanical device for solving non-linear differential equations, J. appl. Phys. 20, 600–607 (June 1949).

The paper introduces an electromechanical device for solving nonlinear differential equations of the form:

$$\frac{d^2\theta}{dt^2} + \psi_1(\theta,t)\left(\frac{d\theta}{dt}\right)^n + \psi_2(\theta,t) = \psi_3(t)$$

in which ψ_1 and ψ_2 are functions of θ and t, ψ_3 is a function of t only, and n is a constant. Examples are given of the results obtained by applying the mechanical analog to the solution of the equation

$$A\,\frac{d^2\theta}{dt^2} + B\,\frac{d\theta}{dt} + (C\theta + D\theta^3) = E\sin\omega t,$$

A, B, C, D, and E being constants. Enrico Volterra, USA

1620. Sh. E. Mikeladze, A new method for the solution of characteristic value problems (in Russian), Doklady Akad. Nauk SSSR 66, 553–556 (1949).

The author applies his method of solving linear differential equations with variable coefficients by reduction to Volterra integral equations [C. R. (Doklady) Acad. Sci. URSS (N.S.) 52, 753–755 (1946)] to characteristic value problems of mechanics. In particular, he discusses the transversely vibrating string with fixed ends and variable density, the linear deflection of a rod with variable axial load and one end free, a beam on an elastic support. Numerical results are given in special cases.
Courtesy of Mathematical Reviews M. J. Gottlieb, USA

1621. Friedrich Ringleb, Numerical and graphical method of conformal representation, Nat. Res. Counc. Canada, Div. mech. Engng., tech. Transl. TT-70, ii + 24 pp. (5 plates) (1948).

[Translated from Zentrale für Wissenschaftliches Berichtswesen des Generalluftzeugmeisters (ZWB), Forsch.-Ber. 1964 (1944).] The first part of this paper is concerned with a modification of Koebe's "Schmiegungsverfahren" for constructing the conformal mapping function which maps a simply connected region on the interior of a circle. The author proposes replacing Koebe's mapping function whenever possible by a function determined in the following way. Assume that the given region R is contained in a circle and that the center of the circle is contained in R. Let a circular arc be drawn which separates the interior of the circle into two regions, a convex region and a sickle-shaped region, such that R is contained in the latter. The mapping function to be used is the function which maps the interior of the sickle on the interior of the given circle subject to a certain normalization. Estimates for the accuracy of the approximations of the mapping functions calculated by this method are not given, although the convergence of the method is proved. The translation is often misleading, e.g., the term "inversion functions" is used when "inverse functions" is meant. The second part of the paper deals with the properties and the construction of small-scale conformal representations.
Courtesy of Mathematical Reviews C. Saltzer, USA

1622. H. Jecklin and H. Zimmermann, A practical interpolation formula (in German), Mitt. Verein. Schweiz. Versich.-Math. 48, 126–144 (1948).

A procedure is given for interpolating from three given values by fitting to them a quotient of two linear functions. This is equivalent to applying Thiele's interpolation formula [L. M. Thomson, *The calculus of finite differences*, Macmillan, London, 1934, chapter 5] as far as second-order reciprocal differences.
Courtesy of Mathematical Reviews T. N. E. Greville, USA

1623. Walter Wuest, The three-point calibration in the design of measuring instruments (in German), Feinwerk Tech. 53, 63–72 (June 1949).

The author develops a simple method of calibrating scales of measuring instruments, especially those used for pressure measurements. The method consists in determining accurately only three positions of the index. In the case of pressure determination, the points are: (1) the position for $p = 0$, (2) for p equal to half pressure, and (3) for p equal to full pressure. Examining and discussing the conditions that have to be fulfilled for the three-point calibration, the author shows how the scale has to be divided starting from the three enumerated positions.
D. DeMeulemeester, Belgium

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 1628, 1637)

1624. O. Bottema, On Cardan positions for the plane motion of a rigid body, Proc. kon. Ned. Akad. Wet. 52, 643–651 (June 1949).

A body in plane motion is said by the author to be in a "Cardan position" at a certain moment if the trajectories of its points all have a momentary third-order contact with those of a Cardan elliptic motion (i.e., a motion in which two points describe straight lines). ("Position" means here really the distribution of the accelerations of the first and second orders.) A few necessary and sufficient conditions for a Cardan position are given in simple terms, and a few theorems about the trajectory curvatures stated.
A. W. Wundheiler, USA

1625. Jules Haag, On the generation of gears (in French), C. R. Acad. Sci. Paris 228, 1989–1991 (June 1949).

To elucidate the geometric and kinematic methods described by J. Capelle in his works [*Théorie et calcul des engrenages hypoïdes*, Paris, 1949, and *Étude de la génération des engrenages par la méthode des roulettes*, Paris, 1938], this note outlines a general analytic approach to the problem of determining methods of generating gear teeth by hobs or other cutting surfaces. In particular, it is shown that even with an arbitrary cutting surface, hypoid gears with linear contact can be cut, in an infinity of ways, by a helical generator movement of the cutter.
Michael Goldberg, USA

1626. Kurt Hain, The generation of given curves by means of gear-crank mechanisms (in German), Feinwerk Tech. 53, 81–89 (June 1949).

The curves traced by points on a connecting rod joining two cranks have been extensively studied. These curves can be modified by allowing curves to be traced by points on gears mounted on the linkage. One or more gears may be mounted with their centers on the pivot points of the four-bar linkage, and simply meshed or meshed in a compound gear train to drive another offset gear. One of the gears is fixed to one of the links. Therefore, the crank curve (a circle) or the connecting-rod curve is modified by an amount proportional to the relative motion of a pair of links. In general, the curve does not close after a complete cycle of the linkage. However, if the linkage makes a num-

ber of cycles (the number is the denominator of the gear ratio), the curve will close. It is easy to design such a mechanism so that a thousand or more cycles of the linkage are made before repetition of the curve begins.

Because of the additional parameters of the mechanism, it is possible to approximate a given curve more closely than with the unmodified four-bar linkage. As many as eight points may be selected on the curve. Graphical constructions are given in which a crank-gear curve or a connecting-rod-gear curve is made to pass through six selected points. Michael Goldberg, USA

1627. **N. G. Chetaev, Concerning the stability and instability of irregular systems** (in Russian), Prikl. Mat. Mekh. 12, 639–642 (1948).

The author considers the perturbed system

$$dx_s/dt = p_{s1}x_1 + \ldots + p_{sn}x_n + X_s, \quad s = 1, \ldots, n \qquad (*)$$

where the coefficients p_{sr} are real continuous bounded functions of t, and the perturbations X_s are holomorphic with respect to the $x_1, \ldots, x_n$ with coefficients which are real continuous and bounded in t. Also, the X_s are assumed to have no constant or first-degree terms in the $x_1, \ldots, x_n$. The following result due to Liapounoff is proved [*Problème générale de la stabilité du mouvement*, Ann. Math. Studies 17, Princeton, 1947, remarks on pp. 251, 254; σ is a positive number defined on p. 251]. If the system of differential equations of the first approximation to (*) is irregular, and if all its characteristic numbers are larger than σ, then the unperturbed motion is stable. A second result is given: if the first approximation system is irregular, and if the least characteristic number is less than $-\sigma$, then the unperturbed motion is unstable.
Courtesy of Mathematical Reviews. E. A. Coddington, USA

Gyroscopics, Governors, Servos

(See Revs. 1629, 1758)

Vibrations, Balancing

(See also Revs. 1651, 1676, 1758, 1762, 1766, 1786)

1628. **R. Koštal, Determination of frequencies and amplitudes of oscillations of coupled nondamped elements** (in Czech, with French summary), Publ. Fac. Sci. Univ. Masaryk no. 300, 25 pp. (1948).

The author starts with n oscillators, the free motion of each of them being governed by an ordinary linear differential equation of the second order with constant coefficients. He then introduces into the system coupling of the most general type where the coupling terms may involve the displacements x_r, the velocities $\dot{x}_r$, and the accelerations $\ddot{x}_r$, of the n particles ($r = 1, \ldots, n$). The system of differential equations governing the motion can be written in the Lagrangian form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}_r} - \frac{\partial(T - V)}{\partial x^r} + \frac{\partial F}{\partial \dot{x}^r} = 0,$$

where T is a quadratic form in the $\dot{x}_r$ plus a bilinear form in the x_r and $\dot{x}_r$, V is a quadratic form in the x_r, and the F is a quadratic form in the $\dot{x}_r$. He investigates the determinantal equation for the characteristic frequencies in particular when $F = 0$, and specializes his result when each element is coupled with its next neighbors only. More special results are obtained for two oscillators with acceleration coupling, and for n identical oscillators when only next neighbors are coupled, and the coupling depends on displacements.
Courtesy of Mathematical Reviews A. Erdélyi, USA

1629. **H. I. Ansoff, Stability of linear oscillating systems with constant time lag,** J. appl. Mech. 16, 158–164 (June 1949).

The author studies a linear oscillating system with a feedback term proportional to the nth derivative of the displacement delayed by a constant time τ. Let t be the time, $\theta\,(t)$ the displacement, I, R, k, constant positive parameters of the system, S the positive magnitude of feedback term. Then θ satisfies the differential-difference equation:

$$I\,\theta''(t) + R\theta'(t) + k\theta(t) = -S\theta(t - \tau)$$

The author proves that harmonic oscillations of the system will exist only for very critical combinations of the parameters I, R, S, k, τ. He shows, by the Nyquist stability criterion, that the system is always unstable for $n > 2$. For $n \leq 2$ the system is stable if certain relations between the parameters are satisfied.
Dario Graffi, Italy

1630. **W. Nijenhuis, A note on a generalized van der Pol equation,** Philips Res. Rep. 4, 401–406 (1949).

The equation $\ddot{v} - \epsilon(1 - 2\beta v - v^2)\dot{v} + v = 0$ is studied by the method of isoclines. For the periodic solution $v = v(t)$, ϵ and β large, it is shown that $-\max v(t)/\min v(t)$ cannot exceed $\frac{1}{3}$.
Courtesy of Mathematical Reviews J. G. Wendel, USA

1631. **W. T. Thomson, Vibration of slender bars with discontinuities in stiffness,** J. appl. Mech. 16, 203–207 (June 1949).

A theoretical method is presented for determining the effect of discontinuities in stiffness on the longitudinal, flexural, and torsional vibrations of slender bars. The discontinuities in stiffness are due to narrow slots or cracks. The results are illustrated by numerical examples. Enrico Volterra, USA

1632. **K. Klotter, Analysis of the various methods for computation of torsional vibrations of machine shafts** (in German), Ingen.-Arch. 17, no. 1–2, 1–61 (1949).

The paper describes in detail the many methods proposed during the past 40 years for solving the torsional natural frequencies of a system consisting of a series of moments of inertia connected by torsional springs. The bibliography shows 33 separate papers, most of which represent different methods of solution: graphical, numerical, by iteration, by trial and error. A list is given comparing the time required to solve a representative system by 11 different methods. J. P. Den Hartog, USA

1633. **J. A. Charnĭĭ, A. J. Freĭdenzon, and Z. T. Arustamova, Dynamic calculation of rods of deep oil pumps considering the friction forces against the pump tubes** (in Russian), Izv. Akad. Nauk SSSR Ser. tekh. Nauk 1949, no. 6, 855–875 (June 1949).

The forced vibrations in a deep-well pump rod are worked out on the basis of a force diagram measured at the upper end of the pump rod. Friction is included, the viscous-damping coefficient being evaluated from the average input-work rate at the top of the rod, and the average output-work rate in lifting a measured quantity of oil from the well bottom. The theory of longitudinal vibrations is worked out to give the loading on the plunger and the motion of the plunger. In applying this theory to an actual oil-well installation, excellent agreement was obtained between the predicted motion of the plunger and the measured output of the well. Walter W. Soroka, USA

1634. **J. R. Forshaw, Vibration measurements on running gas turbines with particular reference to the forces transmitted through the supports,** Proc. seventh int. Congr. appl. Mech. 3, 286–299 (1948).

1635. C. Chu, Centrifugal pendulum dynamic dampers, J. Chinese Inst. Engrs. 6, no. 2, 146–164 (May 1949).

Different types of centrifugal pendulum dynamic dampers are treated, with emphasis on the method of approach and derivation. Oscillations of large amplitude are considered.

Minglung Pei, USA

1636. F. E. Reed, The use of the centrifugal pendulum absorber for the reduction of linear vibration, J. appl. Mech. 16, 190–194 (June 1949).

A device consisting of a centrifugal pendulum is proposed as a dynamic absorber for linear vibrations. The behavior of the absorber at different frequencies is analyzed, and the effects of size, damping, and tuning considered. Curves for evaluating the effectiveness of the device are given, and numerical examples worked out.

Enrico Volterra, USA

1637. G. Gerloff, On a new accelerometer and phenomena of initial vibrations in vibration meters, David Taylor Model Basin Transl. no. 141, 18 pp. (May 1949).

The author discusses the inadequacy of the low-frequency seismic-type vibration meters for the measurement of transient vibrations and accelerations, and shows that satisfactory instruments must have a natural frequency considerably above the highest frequency of the measured shocks.

A new accelerometer is described in which a mass is suspended in three planes on constantan wires 0.002 in. diam. Accelerations of the mass produce proportional changes in the electrical resistance of the wires. These changes can be measured and recorded on electrical instruments of proper sensitivity. Quantitative design and test data are given for such an instrument.

A. Yorgiadis, USA

Wave Motion, Impact

(See also Rev. 1724)

1638. H. Pailloux, Longitudinal impact of a prismatic bar (in French), C. R. Acad. Sci. Paris 228, 2006–2008 (June 1949).

The displacement of a particle on the axis of a prismatical bar with one fixed end and struck longitudinally at the free end may be written as follows:

$$u(x, t) = \frac{4V}{ka} \frac{\sin \alpha}{2\alpha + \sin 2\alpha} \sin \alpha a t \sin \alpha x,$$

where u is the displacement in the x-direction, V the velocity of the striking mass at the moment of impact, k the ratio between the masses of the bar and the striking body, a the velocity of the propagation of extensional waves in the bar, and α the positive roots of the equation $\alpha \tan \alpha = k$. The author introduces the function $\varphi(\lambda) = \sin \alpha\lambda/(2\alpha + \sin 2\alpha)$, and states that with the aid of this function a solution may be found for the impact problem with all possible initial conditions.

R. G. Boiten, Holland

1639. G. Sonntag, Critical considerations of the dynamic resistance of a plate consisting of several layers, stressed by impact (in German), Z. angew. Math. Mech. 29, 157–159 (May 1949).

The author considers two cases of impact stress in a plate consisting of several layers, one impact caused by a large mass of small velocity, and another one caused by a small mass of high velocity. The common basic principles are: the transformation of the kinetic energy into potential strain energy; the expression of the acting momentary force in terms of the mass and deceleration of the impinging body; and the relation between loss of kinetic energy and work of the impinging force along the deflection.

The author investigates the question of whether it is of advantage to divide the plate into several layers in order to reduce the impact force, decrease the deceleration of the point of impact, and thereby decrease the shear stress around the impact center.

Several formulas are given for the central deflection, the bending stress and shear stress in terms of the ratio of impact mass and plate mass, and in terms of stiffness coefficients given by thickness and kind of support. The comparison of the different cases of full and of subdivided plates shows that a thick full plate presents less resistance against dynamic stressing than a plate divided into several thinner plates, a fact which is known already in regard to impact of projectiles against steel plates. If the destruction of the uppermost plates (necessarily of high ductility) is allowed for, then the plates must be spaced so far from one another that kinetic energy is dissipated by plastic deformation.

Coupling of the plates by filling the interspaces between them, for instance with sand, is of advantage for big masses and small velocity, but very disadvantageous for masses of high velocity.

Two previous papers of the author and of W. Fluegge on the derivation of the formulas are quoted.

H. J. Reissner, USA

Elasticity Theory

(See also Revs. 1653, 1656, 1657, 1658, 1661, 1689, 1789, 1796, 1797)

1640. Luigi Broglio, Some synthetic theorems in elasticity and in mathematical physics (in Italian), Monogr. sci. Aero. no. 8, 3–23 (Jan. 1948) = Proc. seventh int. Congr. appl. Mech. 1, 84–97 (1948).

Let y_1 be the actual small deflection of an elastic beam, y_2 the deflection it would have under its actual loading but with fixed ends, and y_3 the deflection it would have under zero loading but with the actual end rotations. Then the author shows that the actual strain energy can be expressed as the sum of the strain energies corresponding to the deflections y_2 and $y_1 - y_2$, and as the difference of the strain energies corresponding to y_3 and $y_1 - y_3$. He formulates a variational principle of equilibrium and develops a method of successive approximations based on these principles. He proposes to extend this method to other problems, but gives no details.

Ed.

1641. H. L. Langhaar, A strain-energy expression for thin elastic shells, J. appl. Mech. 16, 183–189 (June 1949).

Formulas of a rather general nature are derived for the strain energy in a homogeneous, elastic, isotropic shell having radii which are large relative to the thickness. The formula for energy due to bending shows an appreciable deviation from that given by Love [*Elasticity*, 1934] in the first-order terms. The derivation retains, in the strain formulas, terms which are quadratic in the component of displacement which is normal to the middle surface, but neglects terms which are quadratic in the tangential components.

An energy formula is given for flat plates with large deflections. However, for the circular cylinder, elliptic cylinder, and ellipsoid of revolution, the energy formulas are for a small deflection solution, i.e., the displacements are small relative to the thickness of the shell. In the mathematical derivation the lines of principal curvature of the undeformed middle surface are used as an orthogonal curvilinear coordinate system. The derivation is given in tensor notation, and assumes that the reader is familiar with the application of tensor calculus to differential geometry and elasticity.

Stanley U. Benscoter, USA

1642. **R. J. Hank and F. H. Scrivner, Stresses and displacements in a semi-infinite elastic body with parabolic cross section acted on by its own weight only,** J. appl. Mech. 16, 211–212 (June 1949).

The stress distribution in the body is studied, considering it as a problem of plane deformation and determining the Airy stress function. It is proved that there exist a plane surface containing the points of zero horizontal displacement, a curved closed surface containing points of zero vertical displacement, and a plane surface of zero vertical strain. Displacement diagrams for two extreme values of Poisson's ratio are given.

For a cross section with a flat boundary perpendicular to the axis of the parabola the solution cannot fulfill all the conditions in this boundary. Hence, in applying the results to a dam with a rigid or elastic foundation and a parabolic shape, the results are correct only for a limited region (Saint Venant principle). The final results are compared with those for the stress distribution in a semi-infinite elastic wedge. R. G. Boiten, Holland

1643. **A. Thum and O. Svenson, Stresses under the influence of multiple notches. Stress reducing and stress augmenting notches** (in German), Schweiz. Arch. 15, 161–174 (June 1949).

Peak stresses and stress-concentration factors corresponding to a single notch, e.g., hole, fillet, keyway, shoulder, groove, crank, etc., have been obtained in numerous investigations, but there are only few results concerning stresses corresponding to multiple notches. This paper shows how the stress distribution and with it the strength of a structural part are changed by applying multiple notches, rather than a single notch (transverse hole, semicircle notch), and what influence the dimensions of the specimen and notch, the position of the notch relative to the loading direction, and the type of loading (tension, bending) have.

To determine the stress-concentration factors, strain measurements were made, using a specially developed inductive extensometer with a gage length of 0.5–5 mm, a height of 10.4 mm, and a magnification of 300,000. The distribution of stresses was measured for the following multiple notches: series of holes in a thin-walled tube and in a disk; field of holes in a plate; flat specimen with transverse hole and semicircle notch; round and flat specimen with two transverse holes under 90 deg (cross hole); fillet with an oil hole of a crankshaft-crank; and initial point of loading with an additional notch.

The conclusions of practical interests are:

A "file" of notches, i.e., several notches of the same or similar form, reduces the peak stresses and the loading level (stress-reducing notches) corresponding to a single notch preceding them in the load direction.

A "rank" of notches, i.e., additional notches within the notched area arranged across the load direction, may increase, reduce, or leave unaltered the stress-concentration factor according to the form and position of the notches.

An application of additional notches outside of the notched area or a penetrating notch may cause considerable peak stresses (stress-augmenting notches). Max Hempel, Germany

Experimental Stress Analysis

(See also Revs. 1643, 1704)

1644. **R. Pascal, The tensiostats** (in French), Tech. mod. Constr. 4, 171–176 (June 1949).

A "tensiostat" is a device having the property of offering constant resistance over a range of strains or displacements, and is approximated by a slender column pinned at both ends, and loaded axially to the critical buckling load. The author develops the theory of the tensiostat, shows calculations for the design of two tensiostats to operate through a range of deflection under a fixed load, and discusses the application of the concept to the analysis and design of indeterminate structures.

Glenn Murphy, USA

1645. **Edward Wenk, Jr., A frame for testing structural models,** David Taylor Model Basin Rep. no. 599, 22 pp. (Mar. 1949).

This paper describes in detail a testing machine designed to apply simultaneously forces in several directions at many points of large structural models. The loads are applied by hydraulic jacks and are measured with ring dynamometers, on which four SR-4 gages were cemented. The author claims that the deviation from proportionality between loads and strains (read in the indicator used to measure resistance changes in the gages) was less than $^1/_2$% at all loads, and that the shift in the zero in 6 months was less than 15μ in. A. J. Durelli, USA

1646. **H. Lipson and A. R. Stokes, Strengths of metals,** Nature Lond. 163, p. 871 (June 1949).

The note discusses results obtained by Wood and Rachinger on the broadening of X-ray reflection lines from cold-worked metals. The authors state that broadening is due to inhomogeneous internal stresses, not to the smallness of the crystallites into which the grains are broken. Reference is made to previous articles by the same authors on this same topic. D. C. Drucker, USA

1647. **Ernst Mönch, Technique of photoelastic experiments with Dekorit as a model material** (in German), Ingen.-Arch. 16, nos. 3 and 4, 267–286 (1948).

Dekorit is a phenol-formaldehyde plastic made in Germany. By the description of properties given by the author, Dekorit seems to be very similar to Bakelite BT48-306 and Catalin made in the United States.

The author studies thoroughly the mechanical and optical properties of Dekorit, emphasizing the importance of the annealing procedure and its influence on those properties, mainly when the plastic is used in the freezing method. Quantitative values are here given, and the author recommends to anneal for 7 hours at 115 C and to test at 80 C leaving the load on for 2 hours. The reviewer wonders, however, how general the use of these values can be since the original state of the plastic is not well defined and, as is well known, the plastic ages not only artificially by annealing, but also naturally by time. In the last part of the paper the author deals with model similarity and the machining of models.

A. J. Durelli, USA

1648. **Ludwig Foeppl, Slow-motion pictures of impact tests by means of photoelasticity,** J. appl. Mech. 16, 173–177 (June 1949).

In 1891 Saint Venant and Flamant published their theoretical findings of the effect of central transverse impact on the state of stress in a uniform beam simply supported at the ends. They concluded that there exists an interval of time in the initial stage of impact during which the maximum stresses are not at the center of the beam. They further concluded that during this stage the curvature of the elastic curve is not of one sign, so that if the center of the beam is viewed as subjected to a positive bending moment, the rest of the beam toward the ends is subjected to a negative bending moment.

The present paper deals with a photoelastic verification of the above findings, and the author is satisfied that the experiments corroborate the theory. Unfortunately the stress patterns are blurred, vague, and (except perhaps for frame 3) not convincing. From frames 2 and 3 it may be concluded that shortly after con-

fact there develops a marked drop in the stresses at the center, which agrees with the theory.

The reviewer would like to call attention to an aspect of this problem not touched upon in the paper. Frame 3 shows symmetry in the stress distribution about the long axis of the bar. This is at variance with the elastic stress distribution produced by a central, normal, static concentrated load in a long beam. In that state the stress distribution in the center region is not symmetric about the long axis. The pair of isotropic points so characteristic of the static state is here conspicuously absent.

It is not clear whether this is a dynamic effect or the results of imperfections in the technique. Speaking of technique, one wonders why the author chose a beam so thick (0.39 in. for a depth of 0.79 in.) as to rule out a reasonable possibility for a plane stress system. M. M. Frocht, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 1640, 1666, 1685, 1686)

1649. William T. Thomson, Deflection of beams by the operational method, J. Franklin Inst. 247, 557–568 (June 1949).

The problem of deflection of simple beams is solved by means of the operational method. It is pointed out that the operational method has two advantages: (1) In contrast to the classical method which requires equations to be written for each interval between concentrated loads, the operational method enables one to account for any loading by a single equation in terms of boundary values at the origin. (2) Beams with abrupt changes in cross section offer no unusual difficulties to the operational method. Numerical examples are given to illustrate the method.
C. T. Wang, USA

1650. M. Hetényi, A general solution for the bending of beams on an elastic foundation of arbitrary continuity, Proc. seventh int. Congr. appl. Mech. 1, 229–237 (1948).

The problem of the bending of a beam on an elastic foundation has been solved in the past under two extreme assumptions: (1) that deflection of the supporting medium under direct load (without intervening beam) is localized in the loaded area, and (2) that the foundation material is a continuous isotropic elastic body.

To bridge the gap between these extremes, the author in chapter X of his book *Beams on elastic foundation* (Univ. of Michigan Press) has presented a general method for the case of partial continuity. The novel feature of the present paper consists in its application to the particular case of finite beams of arbitrary flexural rigidity, while the book is limited to a solution of the special case of a finite beam of infinite rigidity.

The development is easy to follow and apply, and should be of interest not only to the theorist, but also to practical engineers. The method of analysis should prove particularly useful in the fields of soil mechanics and foundation design, but can be applied to any problem where the elastic supporting medium exhibits a partly local and partly continuous deflection curve under localized pressure. A. R. C. Markl, USA

1651. Th. Vogel, Vibrations of a continuous beam under the influence of a moving load (in French), Ann. Ponts Chauss. 119, 407–424 (May-June 1949).

The title of this paper is not correct. The paper deals with the behavior of a wire that is under tension, has some flexural stiffness, is supported at many equidistant points, and subjected to a moving lateral force of variable magnitude. It was written specifically to apply to the overhead wire and the sliding contact used in furnishing power to streetcars (trams). The problem is first set up in general form (including weight of wire, bending, and tension). Then from the usual orders of magnitude of the various physical quantities in the trolley-wire problem, certain simplifications are introduced into the basic differential equation. The result is to make the bending a secondary correction effect to the basic wire-under-tension problem. Moreover, only three consecutive spans need be considered at any one time: the one with the lateral force, and the two adjoining ones.

The solution consists in finding the normal modes and frequencies of free vibration from the differential equation, and then using Lagrange's equations with the generalized forces corresponding to these modes to get the response. The author shows considerable ingenuity in making transformations and simplifications to obtain useful results. M. P. White, USA

1652. R. P. Eddy and F. S. Shaw, Numerical solution of elastoplastic torsion of a shaft of rotational symmetry, J. appl. Mech. 16, 139–148 (June 1949).

The authors describe, using relaxation methods, an approximate numerical solution for the stress distribution in shafts of rotation symmetry subjected to a torque of sufficient magnitude to cause a portion of the shaft to yield.

Assuming an isotropic material which is perfectly elastic below the yield point, and exhibits perfect plasticity after yield, and also assuming the von Mises yield criterion, the authors give the differential equations of elastic and plastic torsion, and arrive at a set of equations completely defining the problem of elastoplastic torsion. They also show how the common boundary of the elastic and plastic regions can be found if the value of the stress function along the outer contour is known.

The nondimensional form of all equations is also discussed, and a numerical method for the approximate solution of the problem is outlined.

As an example, the problem of a shaft with a collar subjected to torsion is solved for three different values of the torque, the first representing a purely elastic solution. In the appendix the authors give the analytical solution of the elastoplastic torsion of a uniform circular shaft. Nicholas Sag, Australia

1653. Enzo Cambi, Dynamic torsion stresses (in Italian), Atti Accad. Torino 83, no. 1, 43–61 (1949).

The article deals with the determination of the state of stress and strain developed in a shaft due to sudden variation in magnitude of the applied torque. Application of Fourier integrals, Laplace transform, and series of eigenfunctions to the solution of differential equations of the problem are worked out with emphasis on the latter method.

The convergence of the series solution is discussed, a limiting case (when one of the moments of inertia of the masses at the ends of the shaft becomes infinite) is fully developed, and the solution compared with the one obtained by Fourier integrals and Laplace transform. Ernesto Saleme, USA

1654. Che-Tyan Chang, The shape of a piston ring in its unrestrained state, J. appl. Mech. 16, 134–138 (June 1949).

The proper functioning of a piston ring in the engine requires a certain distribution of the radial pressure between the ring and the wall of the cylinder. An even wear both of ring and wall is obtained by a uniformly distributed pressure. But in this case, at high running speeds of the engine, fluttering of the free ends of the ring may occur, inducing fatigue failures close to the ends. To prevent such failures the ring has to exert additional pressure near the ends. The pressure distribution is controlled by the shape of the ring in its unrestrained state.

The author assumes a pressure distribution according to the expression

$$p = p_0 + p_2 \cos 2\vartheta + p_3 \cos 3\vartheta + \ldots + p_n \cos n\vartheta,$$

where ϑ is the polar angle measured from the point opposite the gap. On the basis of this assumption, first the bending moment is calculated. Then, after deriving the known differential equation of the deflection curve of a ring in polar coordinates, the general equation between the free shape of a ring in its unrestrained state and the radial pressure acting on the ring in its close state is developed. Finally the deflection curve of the ring in its unrestrained state is calculated, supposing that the pressure distribution is (1) uniform, and (2) given by the expression

$$p = p_0 + p_2 \cos 2\vartheta.$$

The following assumptions are made: (1) Material of ring follows Hooke's law. (2) Thickness of ring in radial direction is constant and comparatively small in relation to its diameter.

Ferd. Budinsky, Czechoslovakia

1655. **J. A. Haringx, On highly compressible helical springs and rubber rods, and their application for vibration-free mountings, III** (in English), Philips Res. Rep. 4, 206–220 (June 1949).

In part III of this series the buckling and lateral rigidity of rubber columns under axial loading with large compressions are considered, use being made of the equations in parts I and II which deal with helical springs [Rev. 2, **1117**]. Account is taken of the fact that, because of shear deformation, planes initially normal to the center line are no longer so after loading occurs. To calculate the stress-strain relationship for rubber, it is also assumed that the volume of material remains constant. The effects of warping of the cross sections due to shear are assumed to be small for rectangular and circular cross sections. Comparison between the theoretical buckling compressions and the test values obtained by Kosten for rubber columns of circular and rectangular sections indicates that the theory is satisfactory for critical compressions under 50%. The effects of incompressible layers near the ends (where the rubber is vulcanized to the support, for example) are also taken into account. Application of the results is facilitated in the case of circular sections by means of charts giving the lateral rigidity as a function of the axial compression for various ratios of initial length to diameter, both for compressible and incompressible end layers.

A. M. Wahl, USA

1656. **W. Nowacki, The bending of compressed continuous plate,** Proc. seventh int. Congr. appl. Mech. 1, 160–173 (1948).

See Rev. 2, **1121**.

1657. **G. U. Dzhanelidze, Generalized relations of the theory of thin-walled members** (in Russian), Doklady Akad. Nauk SSSR 66, no. 4, 597–600 (June 1949).

This brief paper attempts a generalized treatment of thin-walled members of both open or closed sections subject to compression, bending in two directions, and torsion. The basic equations developed for open sections by Timoshenko, Wagner, Kappus, Goodier, and Vlasov (only the last of whom is mentioned in the bibliography) are rederived by a method of variation of potential energy. They are given in their most basic form only, and in addition are restricted to doubly symmetric sections. The warping function and the quantities connected with it are given in two separate sets of expressions, one for open and the other for closed sections. With these separate expressions the basic equations are held to apply to both types of sections, and to furnish solutions of "any problems in the theory of thin-walled members, including those of stability." Apart from the obvious limitation to doubly symmetric sections, the treatment is too brief and general for this reviewer to form judgment on the validity of this claim and the potentialities of the approach.

George Winter, USA

Plates, Disks, Shells, Membranes

(See also Revs. 1641, 1668, 1669, 1670, 1671, 1673, 1677, 1684)

1658. **A. E. Green, On Reissner's theory of bending of elastic plates,** Quart. appl. Math. 7, 223–228 (July 1949).

The classical linear theory for the bending of elastic plates has been revised by E. Reissner over the past several years to include the effect of the transverse shear deformations. He replaced the fourth-order differential equation by a system of linear differential equations of the sixth order, and the classical two boundary conditions along each edge by three. The system of equations and the boundary conditions were obtained by variational processes.

The present paper shows that Reissner's equations for a plate with shear deformations can be obtained by the use of Stevenson's development of the stress equations of equilibrium and the stress strain relations in complex variable notation. The paper is basically mathematical and contains no applications.

Vito L. Salerno, USA

1659. **H. D. Conway, Note on the bending of circular plates of variable thickness,** J. appl. Mech. 16, 209–210 (June 1949).

In this generalization of his previous study (see Rev. 1, **802**), the author gives general solutions for plates with flexural rigidity $D = D_0 r^m$ (r being the radius, and D_0 and m constants) under uniform loading and under a load uniformly distributed around a central hole. For the latter case he calculates the bending moments and maximum deflection for clamped edges.

L. H. Donnell, USA

1660. **B. R. Seth, Bending of an elliptic plate with a confocal hole,** Quart. J. Mech. appl. Math. 2, 177–181 (June 1949).

An approximate solution is obtained for the bending of a thin elliptic plate with a confocal hole subjected to uniform pressure and clamped at the edges. The numerical results of the deflections obtained are compared with those for a complete plate. It is found that the maximum deflection occurs near the inner boundary. By making the minor axis of the hole vanishingly small, it is pointed out that the case of an elliptic plate with a clamped crack can be treated.

C. T. Wang, USA

1661. **R. Ohlig, The built-in rectangular plate** (in German), Ingen.-Arch. 17, no. 3, 243–263 (1949).

A method is given in complete detail for obtaining the deformations, and therewith the moments and shears, in a rectangular plate fixed along two opposing or all four edges and loaded by an arbitrarily located concentrated lateral force.

The development is an extension of one presented by Happel for a centrally loaded plate fixed along all edges. It utilizes known approaches for solving the basic nonhomogeneous equation for the flat plate: $\Delta\Delta\, w = P/N$, where $N = Eh^3/12\,(1 - \nu^2)$ is the flexural rigidity. For the solution of the supplementary function $w_2 = F_1(x,y) + F_2(x,y)$, which has been introduced to meet the second boundary condition, the device has been employed of substituting one symmetric and three antisymmetric (about the x-axis, the y-axis, and diagonal) loading cases which, when superimposed upon each other, equal the actual loading condition.

This is a rather general solution of the built-in plate problem

and correspondingly the mathematical work required for the solution of a special case becomes quite formidable.

A. R. C. Markl, USA

1662. B. J. Aleck, Thermal stresses in a rectangular plate clamped along an edge, J. appl. Mech. 16, 118–122 (June 1949).

The author discusses the stresses induced by a uniform change in temperature of a thin rectangular plate clamped along one edge. Problems of this kind arise in connection with dam building. The author obtains an approximate solution by assuming the normal stress, σ_x, parallel to the clamped edge, to be of the form

$$\sigma_x = f_1(x) + yf_2(x) + y^2f_3(x)$$

where $f_i(x)$ are determined by the application of the principle of least work. The case of plates where the length of the clamped edge is more than 5 times the length of the perpendicular free edge is taken under special consideration. Formulas and curves are given for the different stresses in this case.

C. J. Bernhardt, Norway

1663. Erik Nilsson, Stress distribution in plates with rotational symmetry and evenly distributed load (in Swedish), Tekn. Tidskr. 79, 485–487 (June 1949).

This paper deals with the bending of uniformly loaded circular plates of nonuniform thickness, where the thickness is a function of the radial distance only. The author shows that an approximate solution of sufficient accuracy may be obtained by dividing the actual plate by concentric circles into a system of ring plates, each of which is given a constant thickness.

C. J. Bernhardt, Norway

1664. G. Sonntag, Stresses in the general, closed or open, conical shell under a load acting on the point (in German), Z. angew. Math. Mech. 29, 178–185 (June 1949).

The author treats the problem stated in the title under the assumptions of membrane theory. The equilibrium equations are satisfied automatically by stress systems which involve arbitrary functions. The author's method is to find these functions by the calculus of variations such that the strain energy is a minimum. This gives a simple solution (for a general cone) for loading by forces but is not successful in giving a closed solution (except for a circular cone) for loading by couples. The latter case may be treated by successive approximations. There are four examples on cones of elliptic and circular cross sections.

The reviewer's opinion is that the stress systems are incorrect. If the cone is slit along a generator and developed on a plane, the stress system will be unaltered. Thus the problem is equivalent to finding plane stress systems in polar coordinates such that $\sigma_{\theta\theta} = 0$. The solutions of the latter problem do not agree with those of the author.

R. C. T. Smith, Australia

1665. D. Williams and M. Fine, Stress distribution in reinforced flat sheet, cylindrical shells and cambered box-beams under bending, Rep. Memo. aero. Res. Counc. Lond. no. 2099, 35 pp. (1940, publ. 1948).

This continues the discussion of stress distribution in the stringers of reinforced sheet, begun in no. 2098, same source, where the simple case of a flat sheet was considered. Ed.

1666. V. Z. Vlasov, Contact problems in the theory of shells and thin-walled box beams (in Russian), Izv. Akad. Nauk SSSR, Ser. tekh. Nauk no 6, 819–837 (June 1949).

The author, a winner of the Stalin premium, discusses the theory of stress distribution in a circular cylinder reinforced by equally spaced stringers and by equal flexible rings. A simplified model is used, assuming that in the axial direction the structure element has no stiffness against bending moments. Thus the equilibrium conditions, reduced to four, take into account only the axial component of the moments. The solution can be developed in trigonometric series, the coefficients of which depend on the axial coordinate. The surface loads, too, are developed in series of cos $n\beta$, β being the angular coordinate. The conclusions, drawn by the author for $n < 1$ and $n > 1$, are very similar to those communicated by P. Cicala [Monogr. Sci. Aeronautica no. 7 (1947)]. The calculation extends also to more complicated cases, where a special section of the cylinder is rigidly chained by a ring of great stiffness. Furthermore the author shows an analogy with the theory of bending of bars, the axial force corresponding thereby to the bending moment.

Walter Wuest, Germany

1667. Wilhelmina D. Kroll and A. E. McPherson, Compression tests of curved panels with circular hole reinforced with circular doubler plates, J. aero. Sci. 16, 354–364 (June 1949).

This paper describes a series of tests measuring the effect of curvature in a panel under compression and containing a circular hole. Specimens tested included plates with the hole both with and without circumferential reinforcement. The results are principally expressed in tables of values of a strain-ratio and a shortening-ratio, comparing the measured strain and diameter-contraction with the uniform strain and contraction in a similar flat plate without hole.

D. N. de G. Allen, England

Buckling Problems

(See also Revs. 1644, 1655, 1666, 1667, 1679)

1668. W. S. Hemp, The theory of flat panels buckled in compression, Rep. Memo. aero. Res. Counc. Lond. no. 2178, 9 pp. (1945, publ. 1948).

This report deals with the post-buckling elastic behavior of plates in single compression, which are supported at the unloaded sides. Using the pertinent von Kármán equations, an approximate solution is obtained by introducing expressions for the deflection w and the stress function U which are periodic in the x-direction, in which the compression works. These expressions consist each of two terms, containing different functions of the variable y. In order to find these functions the expressions for w and U are inserted in the differential equations in three consecutive steps. Formulas are developed for the initial slope of the post-buckled load-strain curve, and numerical results are obtained for simply supported and clamped unloaded edges. The resulting slopes are somewhat smaller than those found from the exact method by S. Levy [Nat. Adv. Com. aero. Rep. no. 737 (1942)].

P. P. Bijlaard, USA

1669. Charles Libove, Saul Ferdman, and John J. Reusch, Elastic buckling of simply supported plate under a compressive stress that varies linearly in the direction of loading, Nat. adv. Comm. Aero. tech. Note no. 1891, 33 pp. (June 1949).

An energy solution is presented for the elastic buckling loads of a simply supported flat rectangular plate subjected to unequal compressive stresses at two opposite edges with a linear variation of stress between those edges. The unbalance is equilibrated by shearing stresses along the other two edges. Compatibility and equilibrium conditions are first investigated throughout the plate, and it is found that these are satisfied if the shear stress varies linearly in the transverse direction and the transverse normal stress is taken to be zero. Minimization of the total energy with respect to each of the Fourier coefficients leads to a

stability criterion in the form of a matrix of order proportional to the number of terms in the assumed deflection series. The solution is directly applicable to idealized cases where the panel forms a unit of the compression surface of, for example, a box girder or airplane wing in bending.

The usefulness of the paper to engineers and stress analysts is greatly increased by the inclusion of charts giving the buckling coefficients for a reasonable range of parameters.

John E. Goldberg, USA

1670. **Henry Favre, The influence of its own weight on the stability of a rectangular plate,** Proc. seventh int. Congr. appl. Mech. 1, 151–159 (1948).

A rectangular plate simply supported on all edges is placed in a vertical position and compressed by a uniform edge load acting in the gravity direction. The energy method is employed by equating the work of the external forces (including the weight of the plate) to the bending energy. Using a double sine-wave series expansion for the buckling displacement, the author calculates the critical buckling load by using only a few terms of the expansion. As a first approximation, it is shown that the influence of the weight of plate on the buckling is that one half of the weight of a vertical element of the plate is added to the compressive edge load. In the remaining work graphical results are given for the critical compressive load as a function of the plate dimensions, the Euler critical column load, and the weight of the plate. Some comparisons are made of the buckling displacements when the weight of plate is included with those when the weight is neglected.

D. L. Holl, USA

1671. **A. van der Neut, Experimental investigation of the post-buckling behaviour of flat plates loaded in shear and compression,** Proc. seventh int. Congr. appl. Mech 1, 174–186 (1948).

See Rev. 2, **995.**

1672. **F. J. Plantema, Some investigations on the Euler instability of flat sandwich plates with simply-supported edges,** Proc. seventh int. Congr. appl. Mech. 1, 200–213 (1948).

See Rev. 2, **724.**

1673. **George Gerard, Symmetrical buckling of sandwich cylinders under compressive end load,** N. Y. Univ. Sch. Aero. Publ., 39 pp. (June 28, 1949).

The buckling problem of a sandwich cylinder is examined by means of the classical method. The effect of a weak core is incorporated in the basic equation. The wrinkling type of buckling is also treated by using a polynomial fitted to the "exact" expression for the foundation modulus given by Hoff and Mautner for a flat sandwich panel. The results of this work may be stated as follows: (a) The critical stress coefficient is essentially constant when $Et/G_cR < 0.1$. (b) When $Et/G_cR \gg 1$ the critical stress is independent of the radius R. (c) The wrinkling stress can be given approximately by the expression $0.826^3(EE_cG_c)^{1/3}$. As a sandwich cylinder can buckle in either of the two types (axisymmetric buckling and wrinkling), an approximate boundary between these two types of buckling is determined. This is given by $h/t = 2.145^3/(G_c/E)^{1/2}$.

Conrad C. Wan, USA

Joints and Joining Methods

(See also Rev. 1712)

1674. **C. Cerardini, Fatigue of welded and riveted trusses,** Weld. Res. Suppl. 14, 241–245 (June 1949).

The article is a translation from Journal de la Soudure (1948). Static and fatigue tests were made on three welded trusses and two riveted trusses. They were all three-panel Warren trusses having a span of 19 ft 8 in. The members of the welded trusses were built up of flat strips. In one case the members were cross-shaped, in another T-shaped, and in the third I-shaped. The members of the riveted trusses were double angles.

Measurements were made for vertical and horizontal movement and rotation of the panel points, strains in the members as well as gusset plates, and for regions of plastic deformation using a special lacquer. Fatigue tests were made at 250 cycles per min. About one million cycles were applied at each of several increments of load. After each of several fatigue fractures, the trusses were repaired and the test continued.

The results of the static tests agreed with calculations, and the action was completely elastic for loads up to 1.5 times the maximum permissible load. The fatigue tests showed the welded trusses superior to the riveted trusses. The secondary stresses in the shallow members of trusses 1 to 4 were of less importance than the defects resulting from poor design and concentration of stress at connections.

Certain conclusions were reached regarding welding: (a) complicated sections should be avoided, (b) longitudinal fillets should be continuous with special transitions at the ends, (c) reduce secondary stress, (d) the greatest importance must be attached to the execution of the welding, and (e) distortion and shrinkage should be minimized by judicious sequence of welding.

Marshall Holt, USA

1675. **K. H. Lenzen, The effect of various fasteners on the fatigue strength of a structural joint,** Bull. Amer. Rly. engng. Ass. 51, no. 481, 1–28 (June-July 1949).

Three groups of structural joints, identical except for the fasteners, were subjected to completely reversed fatigue loading. The fasteners were: (a) hot-driven rivets, (b) cold-driven rivets, and (c) high-tensile-strength bolts. The specimens represented double-strap butt-joints in $^3/_4$-in-thick steel plates with two $^3/_8$-in-thick splice plates. The material conformed to ASTM specification A-7. The fasteners were $^3/_4$-in-diam and the drilled holes were $^{13}/_{16}$-in-diam. The tension-shear-bearing ratios for the main series of tests were 1.00:0.57:0.89.

The main purpose of the investigation was to determine the effect of the clamping force exerted by the fasteners on the fatigue strength of joints. Supplementary studies were made during the fatigue test of the slippage of a joint and of the reduction in the clamping force of bolts. Static tests were made to determine (a) tensile properties of the materials, (b) physical properties of the driven rivets, (c) initial tension in the rivets, (d) the torque-clamping force relation for the bolts, and (e) the effect of plate tension on the clamping force.

The fatigue tests were made in the 250,000-lb capacity structural-fatigue machine at Northwestern University, Evanston, Ill.

The clamping stress developed in the cold-driven rivets was about 3400 psi, and that in the hot-driven rivets about 12,000 psi. The bolts were tightened so as to develop a minimum of 50,000 psi. The fatigue strengths at 2 million cycles were ±14,700 psi, ±15,800 psi, and ±17,200 psi for the cold-driven rivets, hot-driven rivets, and bolts respectively.

The decrease in clamping force in the bolted joints ranged from 0 to 18%. No loosening of the nuts was noted. Static tests with one bolt and also with nine bolts indicated that the coefficient of friction was approximately 0.35.

If sufficiently high clamping force is developed in the bolts, the fatigue failure may occur outside the joint. The crack may not go through holes filled with highly stressed bolts.

Marshall Holt, USA

Structures

(See also Revs. 1639, 1642, 1665, 1667, 1673, 1710, 1759, 1790)

1676. G. E. Hudson, A method of estimating equivalent static loads in simple elastic structures, Proc. Soc. exp. Stress Anal. 6, no. 2, 28–40 (1948).

This paper deals with an application of E. Meissner's graphical solution of second-order differential equations. The structure is a linear elastic system of one degree of freedom. The load is an arbitrary function of time.

The load is replaced by an approximating step-pulse load which is so chosen that in each constant-load time interval the area under the load-time curve equals the area under the actual load-time curve in that interval. The phase graph (Meissner's *Stützkurve*) is composed now of circular arcs. Deflection, velocity, and restoring force can easily be read from the graph.

W. L. Esmeijer, Holland

1677. Bruno Dall'Aglio, On a generalization of Cross's method (in Italian), G. Gen civ. 87: 309–315 (June 1949); 372–375 (July-Aug. 1949).

The author describes an extension of the Hardy Cross method for computing bending moments in plane frameworks in which the joints of the framework are allowed to sway as well as rotate. The author is aware of the general solution of this problem by Grinter, but claims that his method requires much less computation. Part I of the paper starts with a derivation from simple beam theory of carry-over factors for distributing bending moments from a given joint in a rectangular framework to the adjacent joints, when the joints above and below the given joint are free to move horizontally but are not free to rotate. Carry-over factors are also derived for columns which are hinged at the lower end to take care of the case of a framework supported on hinges. The method is then illustrated by determining bending moments at the six joints of a plane framework consisting of four beams and six columns when vertical loads are uniformly distributed over two of the beams. The solution follows the Hardy Cross procedure, differing only in the carry-over factors for bending moments transmitted through columns to adjacent joints above and below. The moments carried over become negligible after three cycles of the computation. In part II of the article the carry-over of moments to the joints is considered for the case in which lateral loads, acting in the plane of the framework, are applied arbitrarily to the columns of the framework. The procedure is again illustrated by solving the same framework as before, in this case loaded by diagonal forces connecting two intermediate points on the framework. The solution converges again after three cycles of computation.

Walter Ramberg, USA

1678. W. Bogucki, Analysis of plane frames by the method of principal points (in French), Arch. Mech. stos. 1, no. 4, 339–364 (1949).

The method of principal points is derived as a technique for solving planar statically indeterminate systems in which the principal action is that of flexure. From the conventional assumptions of elastic members subjected to flexure, the moment at any joint may be expressed in terms of the stiffness of the member, and the rotation at that joint and an adjacent joint. In addition, the rotation at one joint may be evaluated in terms of the rotation at the adjacent joint. The rotations at these joints are proportional to the distances of the joints from a point on the member called the principal point. The location of the principal point is established by the properties of the member, and with the location known the evaluation of moments is simplified in comparison with other techniques.

The author contrasts principal points and "fixed points" and shows both algebraic and graphical procedures for locating the principal points. He also discusses the application of the technique to members having a variable cross section or a curved elastic axis. Several illustrations are included.

Glenn Murphy, USA

1679. J. S. Taylor and R. D. M. Harper, The bending of fuselage shells, Aircr. Engng. 21, 153–158 (May 1949).

⊗1680. Rudolf Saliger, Reinforced-concrete structures: material, design, and construction, 7th ed. (Der Stahlbetonbau, siebente Auflage), Wien, Franz Deuticke, 1949, xiv + 644 pp., 700 figs, 140 tables. Cloth, 6.5 × 9 in. $15.18.

This treatise in reinforced-concrete properties, calculations, and design has been a classical handbook for engineers in German-speaking countries for the last 30 years. In the seventh edition, published after the war, the author expands some chapters, especially those dealing with prestressed concrete and plastic behavior of concrete, to take into account some of the latest developments in these fields. In the reviewer's opinion, however, prestressed concrete properties, techniques of manufacturing and design have not received the necessary attention in the book. Failure to mention some of the latest Belgian, French, and British tests and constructions is particularly noticeable. Appendixes include a large list of bibliographic references and the German reinforced-concrete construction regulations.

A. J. Durelli, USA

1681. Gustave Magnel, Applications of pre-stressed concrete in Belgium, J. Instn. civ. Engrs. 32, no. 6, 161–174 (Apr. 1949).

In this lecture the author reviews briefly the fundamentals of the "post-stressing" method in reinforced-concrete construction, and emphasizes the explanation of the technical details as used in Belgium. The "Belgian cable" is mainly responsible for the extraordinary success of prestressed concrete constructions in Belgium. The author describes some fifty structures built in that country.

A. J. Durelli, USA

1682. T. P. O'Sullivan, The economic design of rectangular reinforced-concrete sections, J. Instn. civ. Engrs. 32, no. 6, 175–206 (Apr. 1949).

This paper deals with the economic factors arising in the design of symmetric reinforced concrete sections under tensile or compressive loads applied normally to the section at a point along one of its axes. The usual assumptions in reinforced-concrete design are made in the calculations.

Ed.

1683. Riccardo F. Baldacci, Contribution to the study of helicoidal structures (in Italian), Att. Ist. Sci. Costr. Univ. Pisa no. 10, 30 pp. (1949).

This paper describes a series of tests carried on helicoidal structures of reinforced concrete under different conditions of support and various load distributions. The experimental results are then compared with the values obtained by a theoretical analysis based on a theory developed by Zanaboni. Finally the behavior of the structure under dynamic loads is determined theoretically and the results compared with experiment.

Ernesto Saleme, USA

1684. Otello Magini, Experiments on large vaults with eliminated thrust (in Italian), Att. Ist. Sci. Costr. Univ. Pisa no. 9, 27 pp. (1949).

The paper describes the tests performed on two shells of reinforced concrete where the horizontal thrust has been eliminated by means of a tension member. The tests results are compared

with a theoretical analysis of the vaults as hyperstatic structures.
Ernesto Saleme, USA

1685. **E. A. F. Huber, Stresses caused by the shrinking of concrete in reinforced concrete structures** (in Dutch, with English summary), Ingenieur 's-Gravenhage 61, Bt 49–52 (June 1949).

The paper deals with the stresses in the concrete and in the reinforcement set up by the prevention of the shrinkage that would occur in unreinforced concrete. Symmetric and symmetrically reinforced sections, and rectangular section beams with only the tension side reinforced are considered. The influence of various factors on the said stresses is discussed, and it is concluded that the shrinkage stresses may be of practical importance.
F. J. Plantema, Holland

1686. **Georg Wästlund and Sigge Eggwertz, Design of concrete pipes** (in Swedish), Betong no. 4, 48 pp. (1948) = Svenska komm.-tekn. Fören. Handl. no. 6 (1949).

The preceding paper is a report of an investigation of the strength of underground conduits, accomplished while working out new Swedish standard specifications for concrete culvert pipes (for highways and railroads) and concrete sewer pipes.

The calculation of loads was carried out according to the common theories of earth pressure which have been adapted and checked by tests for conduits in ditches and embankments by Anson Marston. In designing reinforced-concrete pipes, not only maximum stresses but also widths of cracks were taken into consideration. A width of 0.2 mm at design load for pipes with double reinforcement near the surfaces, and 0.5 mm for single reinforcement in the middle of the barrel thickness, was not considered serious for the permanence of the structure. In order to get some experience and design data for the new reinforced pipes which have a steel content considerably higher than that of pipes in earlier Swedish standard specifications, eighteen specimens of various internal diameters from 500 to 2,000 mm were manufactured and loaded until collapse. First crack load and ultimate load were noted, and deformations and widths of cracks were measured at certain intervals.

Finally some examples of design are given in order to facilitate the use of diagrams and formulas for the reader.
Authors' summary

1687. **John Parmakian, Air-inlet valves for steel pipe lines,** Proc. Amer. Soc. civ. Engrs. 75, 789–794 (June 1949).

In this paper some formulas are given for determining the size and location of air-inlet valves in steel pipe lines, in order to prevent their collapse due to external pressure. Various charts are presented to facilitate the required computations.
R. J. Legger, Holland

1688. **Letterio F. Donato, Tests on a particular complex of foundations for support of electric transmission lines** (in Italian), Att. Ist. Sci. Costr. Univ. Pisa no. 12, 13 pp. (1949).

The paper describes tests performed on the foundations of the towers supporting a high-voltage transmission line.
Ernesto Saleme, USA

Rheology (Plastic, Viscoplastic Flow)

(See also **Revs. 1639, 1646, 1652, 1708, 1716, 1732, 1792, 1796, 1797**)

1689. **Henry Brandenberger, A new theory of elasticity and strength,** Proc. seventh internat. Congress appl. Mech. 1, 14–27 (1948).

The author claims that ordinary elasticity theory "by its first mathematical equation loses all connection between stress and strain," but he adopts the equations of that theory in the familiar form of proportionality of stress and strain traces, and proportionality of stress and strain deviators. He claims (1) that a body never yields under volume changes alone, and (2) that a body yields if any one component of its stress deviator exceeds a certain yield limit, the yield then taking place only in the direction of that component. He thus proposes that each component of the deviator be treated independently and the results superposed, but the reviewer has been unable to find in the paper a statement of the author's stress-strain relations when the yield is exceeded. The reviewer notes that (1) contradicts the common experience that a body ruptures under sufficient hydrostatic tension, and that (2) is not an invariant concept and thus is not admissible for isotropic bodies, although it is possible that the author's vague statement should be interpreted as the Galileo-Navier condition of maximum principal stress, or as a condition of maximum intensity of the stress deviator. The author explains hysteresis and failure. As is customary in the literature of strength of materials, experimental evidence fully confirming the theory is presented.
C. Truesdell, USA

1690. **K. Weissenberg, Specification of rheological phenomena by means of a rheogoniometer,** Proc. int. rheolog. Congr. Holland, II: 114–118; III: 40–42 (1948).

An apparatus for studying the rheological properties of a material is briefly described. The apparatus was developed from a motor-driven precision lathe by means of which the material under examination, held in suitable clamps or containers, could be subjected to a wide variety of types of deformation.

The author makes wide claims for the value of the instrument as a means of determining basic physical parameters of materials, even when these are not rheologically simple, i.e., Hookean solids or Newtonian liquids. In the opinion of the reviewer, any acceptance of these claims should await a complete quantitative study, by means of the apparatus, of a rheologically complex material. Pictures are given in the paper of the production of standing waves in a viscoelastic material.

In the discussion Pollet describes progress in the design of a version of Weissenberg's apparatus adapted to the testing of rubberlike materials. The remaining two contributions to the discussion refer particularly to anomalous properties of viscoelastic liquids which were demonstrated at the conference but are not treated in the paper; e.g., the tendency of certain liquids to rise up a stirrer rotated in them. Reiner poses the question: what is the structure which is responsible for these properties? Rivlin recapitulates the steps in the development of a theory which leads to the conclusion that these anomalous effects could arise from the orientation of molecules in a high-polymer solution subject to laminar flow.
R. S. Rivlin, England

1691. **D. C. Drucker, The significance of the criterion for additional plastic deformation of metals,** J. Colloid Sci. 4, 299–311 (June 1949).

An extension of Prager's mathematical theory of plasticity is given for work-hardening materials where the stresses, as well as the strains and the history of loading, govern the relations between the increments of stress and strain.
J. A. Haringx, Netherlands

1692. **J. H. Palm, Stress-strain relations for uniform monotonic deformation under triaxial loading,** Appl. sci. Res. Sec. A A2, no. 1, 54–92 (1949).

The author develops stress-strains relations for conditions of

biaxial and triaxial stress. These relations are based upon a semi-empirical uniaxial stress-strain equation,

$$\sigma = \sigma_t - (\sigma_t - \sigma_e)\exp[-\epsilon/\epsilon_c] \qquad (1)$$

where σ is the true stress, ϵ the true strain, and σ_t, σ_e and ϵ_c are constants of the material. The following additional assumptions are made: (1) The mechanism of straining can be represented by gliding on the three sets of principal shear planes. (2) The shear strain on each of the principal shear planes depends only upon the value of the principal shear stress. (3) Yielding results whenever the shear stress exceeds a critical value (Mohr-Guest criterion). (4) The three principal strains obey the law

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0.$$

On the basis of these assumptions it is reasoned that the resultant strain for any arbitrary state of stress can be expressed in terms of partial strains, a partial strain resulting from each of the principal shear stresses as follows:

$$\epsilon_1 = f(\sigma_1 - \sigma_2) + g(\sigma_1 - \sigma_3),$$
$$\epsilon_2 = f(\sigma_2 - \sigma_1) + h(\sigma_2 - \sigma_3),$$
$$\epsilon_3 = g(\sigma_3 - \sigma_1) + h(\sigma_3 - \sigma_2).$$

For an isotropic material the functions $f = g = h$, while for an anisotropic material they differ.

The author compares the partial-strain theory with the octahedral stress-strain theory. Some obvious shortcomings of both theories are noted. A more critical comparison of the two theories is then made using the published data based on biaxially loaded tubular specimen tests reported by Maier and by Davis. The conclusion reached is that "the partial strain theory yields results which agree with the available experiments, on the whole, just as well as, or even better than, the octahedral stress-strain theory." An advantage of the partial strain theory results from the fact that it is easily applied and is readily modified for anisotropy of the metal.

Reviewer's note: It should be mentioned that the partial strain theory can be applied without using Equation 1, by using a graphical analysis and the stress-strain curve, or by using some other empirical equation for the stress-strain relation.

William Schroeder, USA

1693. Joseph Marin, J. H. Faupel, V. L. Dutton, and M. W. Brossman, Biaxial plastic stress-strain relations for 24S-T aluminum alloy, Nat. adv. Comm. Aero. tech. Note no. 1536, 96 pp. (May 1948).

Tests on tubes are described for determining the yield stress, nominal ultimate stress, true fracture stress, ductility and elasto-plastic stress-strain relations for 24S-T aluminum alloy when subjected to biaxial tensile stresses. From these tests the authors conclude that the yield stresses are approximately in accordance with the distortion-energy theory, while the nominal biaxial ultimate stresses and the true fracture stresses agree with the maximum-stress theory. The biaxial stress-strain relations in the plastic range are approximated rather well by the so-called generalized Saint Venant theory.

The reviewer remarks that the experimental nominal ultimate stresses and the true fracture stresses are also in accordance with his theory of local plastic deformations [Publ. int. Assoc. Bridge and Struct. Eng. 6, 29–44 (1940)] which is based on the distortion-energy theory, so that the latter holds true everywhere. From his theory in cases where necking is kinematically constrained to occur in the planes of the principal stresses, like in these test tubes, fracture occurs at a maximum principal tension stress which exceeds the fracture stress for unconstrained simple tension by maximum 15.4%.

P. P. Bijlaard, USA

1694. M. P. White, The dynamic stress-strain relation of a metal with a well-defined yield point, Proc. seventh int. Congr. appl. Mech. 1, 329–343 (1948).

In this paper some of the experimental and theoretical results on the travel of stress waves in materials are reviewed. Dynamic stress-strain relations are presented, and the plastic strains which result from these relations are compared with the experimental data. From simplified relations a possible mechanism of deformation under impulsive loading is described for a metal with a well-defined yield point.

M. J. Manjoine, USA

1695. H. De Bruijn, The viscosity of concentrated solutions of polymers, Proc. int. rheolog. Congr. Holland, II: 95–100; III: 36–37 (1948).

A formula is given relating the intrinsic viscosity in the limit of zero concentration to the relative viscosity at elevated concentrations. The formula should be regarded as purely experimental.

H. C. Brinkman, Indonesia

1696. Irving Roberts, Correlation of tension creep tests with relaxation tests, J. appl. Mech. 16, 208 (June 1949).

The author deals with the solution of the equations of stress relaxation: (1) $\epsilon_e + \epsilon_p = \epsilon_{eo}$ (e refers to the elastic, p to the plastic, o to the state at time $t = 0$); (2) $\epsilon_p = \sigma/E^3$;

$$(3)\ \epsilon_p = (c/E)[\exp(\sigma/c) - 1]\cdot T$$

(c is a constant, T a monotonically increasing function of t, (3) is derived from creep tests). He points out that this solution may be directly obtained by substituting (2) and (3) into (1):

$$T = (\sigma_0 - \sigma)/c[\exp(\sigma/c) - 1],$$

and that it is not necessary first to form a differential equation as Soderberg, Popov and Housner [Trans Amer. Soc. mech. Engrs. 58, p. 733 (1936); 69 A-135 and A-352 (1947)] have done.

Albert Kochendörfer, Germany

1697. W. Boas, The interaction between the crystals of an aggregate during plastic deformation, Proc. seventh int. Congr. appl. Mech. 1, 356–364 (1948).

A discussion is given of the main assumptions made by G. I. Taylor in his calculation of the stress-strain curves for polycrystalline materials (of face-centered cubic metals) based on those for single crystals. From observations described, it is concluded that the extent of plastic deformation can vary within crystals, and from crystal to crystal, but is continuous at grain boundaries.

H. G. Hopkins, England

1698. R. S. Brown, Plastic strain and hysteresis in drawn steel wire, J. Iron Steel Inst. 162, 189–200 (June 1949).

A review of various types of testing drawn-steel wires under static and dynamic stresses is presented. Numerous graphs show the stress-strain relationship due to static and repeated loads, and for various chemical compositions, diameters, heat treatments, and aging of the wires. Permanent set, hysteresis, and creep characteristics are tabulated. The effects of core wires in locked-coil winding ropes and wires for prestressed reinforced concrete are discussed.

Main conclusions for durable rope wires concern high plastic and hysteresis properties. For future improvement, complete suppression of temperature increase after the drawing process is suggested.

R. K. Bernhard, USA

1699. **Tadasi Isibasi, Plastic failure of poly-crystalline materials,** Memo. Fac. Engng. Kyushu Univ. 9, no. 4 (1949).

The yielding of a solid body is interpreted as a kind of change from the normal state to a state where slip band appears. To discuss this change of state, a model is proposed consisting of a slip band from the surface to a small imaginary hole, with a dislocation corresponding to the amount of the slip. By doing virtual work on this model, an equation showing the relation between yield point and the length of a slip line is obtained. This equation shows the importance of the stress condition of a second point distant from the point under consideration. It is shown that by this theory satisfactory explanation of many experimental facts concerning yield points of specimens under nonuniform stress distribution is possible. Also the scale effect is explained by this second-point theory.

Torsion of round bars, thick cylinders under internal pressure, and centrally perforated plates in tension are accounted for.

The reviewer should like to state that the assumption that yielding lines are slip lines is not quite compelling and in some degree is contradicted by the existence of curved, noncircular, yielding lines.

Also the scale effect may be explained by the influence of surface finish, or by a statistical theory corresponding to Weibull's theory of rupture. K. W. Johnsen, Denmark

1700. **J. J. Hermans, Some aspects of swelling,** Proc. int. rheolog. Congr. Holland, II: 179–196; III: 58–59 (1948).

In the first part of the paper, it is pointed out that the thermodynamical formulas which describe the swelling of gels are particularly simple when expressed in terms of force and displacement, instead of stress and strain.

In the second part a theory of anisotropic swelling is given. It is assumed that a gel is built up of anisotropic elements for each of which the tendency to swell in a specified direction is different from that in others. The orientation of adjacent elements is different, thus hindering the swelling by steric reasons. It is then assumed that the gel behaves as an isotropic body with respect to stresses. These stresses are assumed to be proportional to the difference between the actual strains and those occurring in the absence of steric hindrance. As a result differential equations are obtained describing the swelling process as if the gel were a nonswelling solid subject to volume forces and surface forces.

The change in over-all dimensions conforms to a formula given earlier by P. H. Hermans.

Finally, it is shown that if the orientation of the structural elements is no systematic function of the spatial coordinates, and if the orientation is not correlated to its gradient, then the over-all orientation is not affected by the swelling process.

H. C. Brinkman, Indonesia

Failure, Mechanics of Solid State

(See also Revs. 1648, 1693, 1697)

1701. **R. C. Grassi and I. Cornet, Fracture of gray-cast-iron tubes under biaxial stresses,** J. appl. Mech. 16, 178–182 (June 1949).

Thin-walled tubes of gray cast iron were tested under biaxial stresses to determine the validity and applicability of the present theories of fracture. The specimens were stressed according to the desired "stress ratio" (i.e., ratios of axial to tangential stress) by the proper combination of internal pressure and axial tension or compression. The results cover the determination of the true stresses, type of fracture, and ductility. Further tests under combined stresses on such brittle materials as gray cast iron are necessary, because the results of the present investigation cannot be described exactly by the different theories of fracture.

M. Hempel, Germany

1702. **R. L. Stoker, Erosion due to dust particles in a gas stream,** Indust. Engng. Chem. 41, 1196–1199 (June 1949).

In an attempt to solve certain erosion problems associated with fluid-type catalytic packing plants, experiments were made to determine rates of erosion caused by granular materials or dusts blown at surfaces under relatively high velocities. Three dusts were used. One of these was a fine silica sand blown against the target in an air stream with velocities ranging from 100 to 600 fps, and the other two were different grades of synthetic catalyst blown with velocities of 60 to 150 fps. The target materials used were black iron and gypsum plaster. Plaster was used in an attempt to work out a standard test for conveniently predicting erosion life of more durable materials. Most of the experiments were made with the blast striking normal to the target surface, although in one series with the black iron target, the angle of impingement was varied from about 20 deg to 90 deg. The results indicate that erosion rates rise very rapidly with velocity of the blast, and that there is a critical angle of blast at which the erosion rates are a maximum. Vito A. Vanoni, USA

Design Factors, Meaning of Material Tests

(See Rev. 1648)

Material Test Techniques

(See also Revs. 1645, 1648, 1698, 1701, 1708, 1713, 1715, 1792, 1798)

1703. **A. Képès, A new method to measure Young's modulus and the internal friction of solids,** Proc. int. rheolog. Congr. Holland, IV, 66–72 (1948).

The experimental method proposed is the rotating cantilever, as used by Kimball and Lovel (Phys. Rev. 30, p. 948, 1927). The specific damping capacity of two vinyl chlorides is found to peak at 370 and 580 cpm, while that of ebonite decreases and that of Saran increases with frequency. J. M. Robertson, USA

1704. **R. J. Hansen, Controlled impulsive-load testing machine,** Proc. Soc. exp. Stress Anal. 6, no. 2, 64–67 (1948).

The subject testing machine and its auxilliary instrumentation are carefully described. The time for the load build-up is approximately 0.001 sec. The duration of the load may be varied from a lower practical limit of 0.01 sec to 0.5 sec. A mechanical timing device is used to initiate and to control the various events in a test. The force-generating system is a high-pressure cylinder and a light piston and piston rod. The load is applied by a trigger arrangement on the piston rod. The load is released by puncturing an aluminum diaphragm, thus permitting a rapid escape of the air from the high-pressure cylinder. George H. Lee, USA

1705. **A. Pogany, Determination of deformations and stresses in pipes under earth fills on the basis of investigations on models,** Proc. seventh int. Congr. appl. Mech. 1, 297–307 (1948).

The author has executed tests on small paraffin models which deform more than the original construction under the same conditions, according to the principles of modeling. The influence of this fact on soil reactions is, however, disregarded. The stresses under the pipes are considered uniformly distributed, and a bedding constant is introduced to avoid complicated computations in determining the reactions of the material around the pipes. Results of tests are given. F. C. de Nie, Holland

1706. M. A. Fisher and E. F. Davis, Studies on fly-ash erosion, Mech. Engng. 71, 481–487 (June 1949).

The erosion of metals through the impingement of solid particles suspended in a rapidly moving gas stream has been investigated for studying the possibility of serious erosion of turbine blades and other metal parts by the suspension of fly ash in hot combustion gases. The erosion tests were carried out at room temperature as well as at high temperature (800 F to 1350 F). The rate of metal erosion by unseparated fly ash at room temperature increased with increasing jet velocity; the rate of increase was especially high at jet velocities approaching sonic velocity. The fly-ash concentration in the air stream was kept between 2 and 3 grains per cu ft. In contrast to the erosion tests carried out with raw fly ash at room temperature, when an Aerotec tube was used to remove the coarser particles of fly ash, a persistent deposit of ash particles was formed on the surface of the erosion specimen. This deposit protected the surface against erosion. The chief feature of all erosion tests at high temperature was the formation of hard tenacious coatings or deposits on the specimens. This deposit formation was associated with the impingement of relatively fine ash particles, smaller than about 10 microns diam, while coarser particles tended to remove any deposits present on the specimen face and to produce erosion of the metal.

W. Kochanowsky, Germany

Mechanical Properties of Specific Materials

(See also Revs. 1643, 1655, 1674, 1701, 1703, 1706, 1783, 1811)

1707. C. A. Wert and E. P. T. Tyndall, Elasticity of zinc crystals, J. appl. Phys. 20, 587–589 (June 1949).

The modulus of elasticity E was measured by the authors by dynamic and static methods for 25 single zinc crystals. The results fitted a parabolic curve when $1/E$ was plotted against the square of the cosine of the orientation, as predicted by theory. From the constants of this curve and earlier data on linear compressibility by Bridgman, the authors derived numerical values for the coefficient of compliance of the zinc crystal, as follows: $S_{11} = 8.38$, $S_{12} = 0.5$, $S_{13} = -7.31$, $S_{33} = 28.4$, $S_{44} = 26.1$, all in units 10^{-13} cm²/dyne.

Data on three crystals were taken from room temperature to 375 C. From these data the modulus of elasticity may be computed for any orientation and any temperature in the range given. The authors observed that the modulus of elasticity decreased for all orientations with increasing temperature.

William N. Findley, USA

1708. Charles Appert and Robert Cabarat, A study of the hardening of a light alloy from an anelastic point of view (in French), C. R. Acad. Sci. Paris 228, 1871–1873 (June 1949).

Reports measurements of internal friction (δ decrement) as a function of aging time for a commercial, high-strength Zn-Al based alloy (similar to 75S, Zicral, T.60). The experimental method [same source, 217, p. 229 (1943)] involved a longitudinally vibrating specimen supported at nodes in a vacuum (frequently not stated). Solution heat treatment of the cylindrical specimens was 30 min at 450 C, quenched and aged at 18 ± 1 C. δ was found to increase sharply with aging time from about 0.5×10^{-4} up to a maximum of approximately 3.2×10^{-4} at about 4 hr aging. For greater times, δ decreased. A significant, sharp drop in δ occurred for aging times between 100 and 200 hr. Subsequent aging (up to 1000 hr maximum) brought about further reduction at a rate comparable to that observed between 4 to 100 hr aging. Normal changes in Young's modulus, Rockwell hardness, and electrical conductivity were also observed. The authors suggest that the initial rapid rise in δ which occurred prior to the development of maximum hardness is associated with that phase in the development of the precipitate particles during which the lattice of the particles is essentially coherent with that of the locally strained matrix.

Waller George, USA

1709. Gerhard Schikorr and Günter Wassermann, On the influence of natural weathering on the stress-corrosion of aluminum alloys (in German), Z. Metallk. 40, no. 6, 201–205 (June 1949).

The paper summarizes the results of a study of the atmospheric stress-corrosion cracking of several high-strength aluminum alloys at seventeen different locations in Germany. To insure consistent results care must be exercised to bend the loops uniformly, as sharp radii cause premature breaks. Only occasional stress cracking, within a period of one year, was observed with an aluminum-copper-magnesium alloy, an aluminum-magnesium (9%) alloy, and an aluminum-zinc-magnesium alloy that contained vanadium and chromium. The latter type of alloy, without vanadium and chromium, was highly susceptible to stress cracking after water quenching, the half time for cracking 50% of the specimens being between 10 days (near the seacoast) and 105 days (country). This alloy could be improved considerably by a special (not described) heat treatment, the half time then being nearly one year or more (i.e., only a few specimens cracked within a year). Protection from rain increased the tendency to stress crack. In a 70 C NaCl solution, the quenched aluminum-zinc-magnesium alloy specimens broke between 30 to 65 min, the specially heat-treated specimens of this alloy between 25 and 116 hr, while those of other alloys did not crack within 50 days.

George Sachs, USA

1710. Frank R. Beyer, Stresses in reinforced concrete due to volume change, J. Amer. Concr. Inst. 20, 713–722 (June 1949).

A procedure is described for measuring, with the aid of SR-4 strain gages, the stresses developed in the steel of a reinforced-concrete specimen as a result of volume changes of the concrete (shrinkage) in the course of the hardening process. The observations start after the pouring and extend to the first 64 days, readings being taken at very short intervals at the beginning of the test so that a clear picture of the changes in stress in the course of the first 90 hr is obtained. The existence of the shrinkage stresses both in the steel and the concrete is well known; actual observation of their magnitude has, however, been rather infrequent.

A. M. Freudenthal, USA

1711. Riccardo F. Baldacci, Statistical research on the resistance of the concrete no. 500 (in Italian), Att. Ist. Sci. Costr. no. 14, 12 pp. (1949).

This paper deals with a statistical analysis of tensile and compressive cement-mortar tests. The author found that in both cases the standard deviation of specimens 7 days old is essentially the same as the standard deviation of specimens 28 days old. Recommendations are made to introduce changes in the interpretations of concrete specifications. The values of m and M on page 5 are erroneously given as 10 units too large.

A. J. Durelli, USA

1712. H. W. Russel and Eivind Hognestad, Effect of entrained air on the bond strength of concrete, Highw. Res. Bd. Proc. 28, 195–210 (1948).

Pull-out tests in which bars were pulled from concrete cubes were performed considering the following variables of the tests: three air contents, 0, 4, and 6% of air, three positions of casting, and two types of reinforcing bars of two different diameters. Three specimens were used for each variable. Slip between the

bar and the concrete was measured at both the free and the loaded end while continuing the loading until the bar had passed its yield point.

The results show that for both types of bars, when cast vertically, the bond for a specified slip is reduced by 10% at 4.5% of air content. This reduction was about the same as the corresponding reduction in compressive strength and modulus of rupture. For entrainment above 4.5% the results for the two types of bars were different; the bond will, however, generally be reduced less than the compressive strength and modulus of rupture. For the horizontally cast bars 4% of entrained air caused practically no change in bond, while for air entrainment over 4% the bond was reduced as well as the compressive strength.

Not all the variables which may affect relations between air content and bond were included in these tests, as for instance the water-cement ratio of the concrete and grading of aggregates. Also no comparison has been made with the behavior of plain reinforcing bars under similar conditions.

This reviewer believes that the conclusions arrived at are not wholly justified, especially as some tests show rather large scattering. M. Reiner, Israel

1713. **R. E. Bollen, Durability of Portland cement concrete determined by primary directional freezing and uniform thawing,** Highw. Res. Bd. Proc. 28, 218–232 (1948).

Concrete beams 3 × 4 × 16 in. were laid directly on freezer plates and frozen. Thawing was accomplished by continually circulating steam over the beams, the temperature range being between 70 F and −30 F.

Time-temperature records are presented for different heights of the beams above the freezing plates, indicating that five cycles of freezing and thawing per day can be carried out if the beams are placed directly on the freezing plates. This is supposed to produce a degree of disintegration similar to that produced by the conventional method in less than one tenth the number of cycles, and at the same time resembles more the actual conditions of freezing on many highway pavements.

Length changes, sonic modulus, flexural strength, and compressive strength were determined on each specimen at five-cycle intervals, but no correlation between the conventional tests or field tests was worked out, owing to limited data. The curves show marked differences in accordance with different ages and curing conditions of the concrete. M. Reiner, Israel

1714. **S. W. Marsh, Rubber as a stress-carrying material and some design considerations,** Proc. Instn. mech. Engrs. part II, 25–41 (1948–49).

The main paper and its numerous discussions represent a general but somewhat practical explanation of the behavior of rubber under various types of mechanical load. Preloaded and bonded type of applications are discussed together with some of the numerous engineering uses and limitations of rubber.

A. Yorgiadis, USA

1715. **J. W. Ballou and J. C. Smith, Dynamic measurements of polymer physical properties,** J. appl. Phys. 20, 493–502 (June 1949).

A description is given of three techniques suitable for investigation of dynamic elastic and dissipative properties of high polymers in fiber and film form in the frequency range from 3–30,000 cps. It includes a discussion of effect of frequency on the parameters, and applies the techniques to the study of stiffness-temperature behavior of high polymers such as 66 nylon, polyethylene terephthalate, and polyethylene.

Dynamic tests involving small strains are treated by reduction to an equivalent system composed of a mass, spring, and dash-pot. The frequency range is covered by using transverse waves in a stretched string for the high range, flexural beam resonance (reed) for the middle range, and longitudinal vibration of a long string with a heavy mass at the end for the low range. Heating effects are found by converting power dissipation to heat units, assuming no conduction or radiation losses from the sample. The results stated are that the dynamic modulus increases over the frequency range covered, while internal viscosity coefficient and relaxation time decrease. Relaxation time is found to depend more on chemical structure than the physical state. An appendix gives derivation of measurements of small acoustic attenuation in fibers. Thomas A. Hunter, USA

1716. **J. A. Sauer, J. Marin, and C. C. Hsiao, Creep and damping properties of polystyrene,** J. appl. Phys. 20, 507–517 (June 1949).

The paper describes an investigation of the anelastic behavior of large-sized specimens of polystyrene cut directly from moulded sheets. The tensile creep tests carried out under long-time load application and at various stress amplitudes revealed a linear relation between the logarithm of the creep rate and that of the stress amplitude. The damping behavior was studied by the resonant vibration method first developed by Lazan [Trans. Amer. Soc. mech. Engrs 65, p. 87 (1943)] who found that a similar relation as above existed. As both creep and damping are manifestations of anelasticity, creep properties could be advantageously derived from studies of the behavior of damping, usually simple to carry out and involving only short-period tests. The data indicate that the same molecular mechanism is responsible for both types of behavior.

The authors also discuss the phenomenon of "crazing," in the light of theories put forward by Maxwell, Rahm, and Russel, in addition to their own observations. They conclude that crazing which occurs in creep specimens is due to an orientation of ordered regions of the molecular structure in the direction of the applied stress. This interpretation is also confirmed by X-ray spectrometer tests. S. K. Ghaswala, India

1717. **Fred Werren and C. B. Norris, Directional properties of glass-fabric-base plastic laminate panels of sizes that do not buckle,** For. Prod. Lab. Rep. no. 1803, 49 pp. (April 1949).

This paper presents a study of the elastic and strength properties of glass-fabric laminates. Experimental results are given and compared with values calculated from the theory of orthotropic materials, i.e., materials having three mutually perpendicular planes of symmetry. These comparisons are presented for tests in tension, compression, and shear on specimens so proportioned as to preclude buckling or failure because of elastic instability.

The material is comprised of glass fabric of 181–114 satin wave of 25 plies each, parallel laminated with a high-temperature-setting, low-viscosity, laminating resin of the alkyd styrene type polyester. Proportion of resin by weight was approximately 36%.

Experimentally determined stress-strain curves in tension, compression and shear for various angles of the grain of the fabric are included. Curves of tangent modulus as a function of tensile stress, and curves of modulus of elasticity, proportional limit, and ultimate stress as a function of angle of the grain are included. The latter curves show a comparison between theoretical and experimental values. Charles E. Crede, USA

1718. **Fred Werren, Effect of span-depth ratio and thickness on the mechanical properties of a typical glass-fabric-base plastic laminate as determined by bending tests,** For. Prod. Lab. Rep. no. 1807, 6 pp. (1949).

Three thicknesses of glass-fabric-base plastic laminate were tested in bending, compression, and tension in order to investigate the effect of span-depth ratio and thickness on the mechanical properties.

The three thicknesses used in the investigation were $^1/_{16}$, $^1/_4$, and $^1/_2$ in. Seven plies were used for the $^1/_{16}$-in. sheet, 25 for the $^1/_4$-in. sheet, and 50 plies for the $^1/_2$-in. sheet. The fabric was parallel-laminated with a high-temperature-setting, low-viscosity, laminating resin.

The author arrives at the following conclusions: (1) As the span-depth ratio for a given thickness of laminate is increased, the resultant value of modulus of rupture is decreased. (2) The rate of decrease of modulus of rupture resulting from an increase in span-depth ratio appears to become less as the thickness of the laminate is increased. (3) For a given span-depth ratio, a thin laminate has a higher modulus of rupture than a thick one. Additional studies are needed to investigate the cause of the variation of mechanical properties of laminates with thickness. (4) The modulus of elasticity appears to be independent of the thickness of the laminate, and increases slightly with an increase in span-depth ratio, probably due to decreasing effects of shear. (5) On the average, the fiber stress at proportional limit for a given thickness of laminate decreases with an increase in span-depth ratio. Karl Arnstein, USA

1719. F. H. Müller, Stress-strain diagrams with interpolated analysis, at various extension rates, for different single organic fibers (in German), Kolloid Z. 113, no. 3; V: 159–165, VI: 116–170 (June 1949).

The apparatus and instrumentation used in this investigation were described in a previous article [same source, 112, p. 84 (1949); Rev. 3, **486**]. Part V gives the results of a series of cyclic stress-strain tests made on the single fibers of such organic materials as wool, cotton, etc. These cyclic tests, made at several extension rates, were made with the objective of obtaining relaxation data. The results of these tests were analyzed in an attempt to break down the total strain into its component parts, elastic and plastic. For the most part, it was found that the plastic strain far exceeded the elastic strain; stretched Perlon at the highest rate of extension is the only exception found. Pe-Ce and wool demonstrated definite elastic limits.

Part VI considers the effect of temperature on the stress-strain characteristics of such materials as Pe-Ce, Polyvinyl alcohol, Perlon, etc. The stress-strain diagrams indicate both the nominal and the true stresses. The results indicated a general decrease in Young's modulus of elasticity and an increase in the ultimate strain with increasing temperature. A lowering of the ultimate strength with increasing temperature is indicated; the magnitude of the change in the ultimate strength varies greatly from material to material. George H. Lee, USA

Mechanics of Forming and Cutting

1720. G. A. Oding, Heating in cutting process (in Russian), Doklady Akad. Nauk SSSR 66, no. 4, 585–587 (June 1949).

The author reports his results from a study of the effect of the cutting speed, feed and depth of cut on the temperature of the work when turning plain carbon steel with a carbide-tipped single-point cutting tool. The temperature of the work was determined by inserting a thermometer in the central part of each cylindrical specimen, and by recording the maximum temperature rise. Unfortunately the author does not give any detailed description of his experimental setup.

The results which are shown in the form of three sets of graphs, and the brief commentaries made by the author yield the following conclusions: all the other conditions remaining the same, the work temperature increases with an increase of the depth of cut, and it decreases with an increase of the feed or the cutting speed; the effect of feed is much more pronounced than the effect of the cutting speed. R. A. Galuzevski, USA

Hydraulics; Cavitation; Transport

(See also Revs. 1687, 1724, 1730, 1755, 1769, 1782, 1801, 1802, 1803, 1805, 1809)

1721. Ahmed Shukry, Flow around bends in an open flume, Proc. Amer. Soc. civ. Engrs. 75, 713–741 (June 1949).

The flow around bends in an open flume was studied by means of a Pitot sphere of the type commonly known as the van der Hegge Zijnen gage. Velocity and energy distribution were measured in several types of bends, and the spiral motion analyzed. The energy lost due to bend resistance varied considerably with Reynolds number, curvature, depth of flow, and angle of bend (no consideration was given to the action of roughness). This variation explains differences in uncorrelated experimental results.

Some consideration was given to beds formed of erodible material. A radius of bend equal to 3 times the width is recommended as giving the minimum radius at which all the bend action effects are negligible. André L. Jorissen, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 1618, 1740, 1754, 1755, 1773, 1810, 1812, 1814)

1722. G. Birkhoff, Recent developments in free boundary theory, Proc. seventh int. Congr. appl. Mech. 2, part I, 7–16 (1948).

Recent developments in overcoming the three defects in the study of flows with free boundaries—mathematical indeterminacy, two-dimensionality, and physical unreality—are described.

In the introduction it is pointed out that the density ratio ρ'/ρ across the surface of discontinuity and the cavitation number $K = (p - p')/\frac{1}{2}\rho v^2$ (p' is the pressure on the free surface, p the pressure of the undisturbed flow, and v' and v are the corresponding velocities) are physically important. Some recently constructed flows with $K \neq 0$ are then mentioned. A discussion is given of recent work on the problem of determining which of the possible mathematical jets that can issue from a nozzle, and which of the possible mathematical wakes that can follow a given two-dimensional object occurs. Progress in the theory of axially symmetric flows with free streamlines is discussed. A summary is presented of the relation between the theory and the evidence obtained from the study of photographs of cavities formed behind missiles shot into water at high speeds, and of cavities formed behind obstacles held in a water tunnel. (Reviewer's note: The dependence of cavity shape on the type of boundary layer that forms on an object that enters water from the air is stressed in the paper, "Boundary layer effects on spinning spheres," by H. Wayland and F. G. White, presented at the 1949 meeting of the Heat Transfer and Fluid Mechanics Institute.)

It is stated that the corrections to the drag are large for fully developed cavity flows bounded by walls, but that the corrections can be made negligible by using the downstream velocity v' instead of the upstream velocity v to compute the drag coefficient. Finally, the effectiveness of lined-cavity charges as antitank

weapons is explained by the classical theory of impinging jets, and a mathematical model for the penetration of armor by a jet from a wedge-shaped liner is included. Neal Tetervin, USA

1723. **Jean-Jacques Moreau, On vortex interpretation of slipstream surfaces** (in French), C. R. Acad. Sci. Paris 228, 1923–1925 (June 1949).

The author's objective is to determine the change of the surface curl upon a vortex sheet as a motion proceeds. He states a kinematical formula for its derivative relative to a point fixed with respect to a surface coinciding with the vortex sheet and endowed with a velocity which is the mean of those upon its two sides. From this formula he concludes that in barotropic motions of an inviscid fluid subject to conservative extraneous force there are theorems for the surface curl which are direct analogues of those of Helmholtz and Kelvin for continuous motions. Since the author employs the false assumption that for barotropic motions of an inviscid fluid there is no tangential discontinuity in the acceleration (one of many counter examples:

$$u = kxt,\ v = 0,\ w = 0,\ \rho = \text{const, extr. force} = 0,$$

$$p = -\rho k(1 + kt)x^2/2,\ k = k(y) = 1 \text{ if } y > 0, = 0 \text{ if } y < 0),$$

the reviewer feels some hesitance in accepting the result.

C. A. Truesdell, USA

1724. **Robert F. Dressler, Mathematical solution of the problem of roll-waves in inclined open channels,** Commun. pure appl. Math. 2, 149–194 (June-Sept. 1949).

Roll-waves are a characteristic occurrence of high-velocity flows in chutes. The author shows first that no continuous solution can be found for this type of flow based on the shallow-water theory. The similarity of the roll-wave profile with a combination of backwater curves and hydraulic jumps lead to the following solution: While the standard back-water curves as solutions of the Bresse equation represent steady-flow patterns in a uniform channel, similar stable-flow patterns can be derived which move with a constant velocity along the channel. This means that the shape of the water surface moves along the channel with a wave velocity which is different from the flow velocity of the water. The shapes of these water-surface lines are different from the solutions of the Bresse equation because of the nonlinearity of the friction term with respect to the flow velocity. Hydraulic jumps, moving with the wave velocity, can also be calculated. The roll-waves are described finally as a sequence of such continuous water-surface lines and hydraulic jumps moving downstream at an equal wave velocity. The method is significant as a generalization of the Bresse equation. H. A. Einstein, USA

1725. **Pierre Theron, On a theorem of existence of plane fluids movements with wakes** (in French), C. R. Acad. Sci. Paris 228, 1922–1923 (June 1949).

Considering a plane symmetric flow of a perfect fluid, the boundary being subjected to very general conditions, the author shows the existence of wakes starting from the contour and not cutting it again. This result, solving also the problem of the prow, constitutes a new extension of the work of Henri Villat.

L. Escande, France

1726. **A. S. Peters, A new treatment of the ship wave problem,** Commun. pure appl. Math. 2, 123–148 (June-Sept. 1949).

The paper is concerned with the determination of the surface waves created by a concentrated pressure which moves in a straight course over the surface of a body of water whose depth is great. This is an important problem because, as a first approximation, the moving pressure point can be considered as representing the bow of a moving ship.

The free-surface shape is very d fficult to derive. It depends upon a potential function which satisfies certain nonlinear boundary conditions at the surface, whose shape is not given but must be determined. A linearized approximation has been formulated and solved by Kelvin under the assumption of small wave amplitude and small difference between the velocity of the pressure point and water.

In this paper a new representation for the solution of the linearized problem is obtained. The approach used is based upon an extension by Stoker [Rev. 1, 179] of a method by H. Lewy [Bull. Amer. math. Soc. 52 (1946)], which solved two-dimensional problems of waves on sloping beaches by using functions of a complex variable in a special way. The results are interesting to compare with those originally obtained by Kelvin. Kelvin found that the surface-wave amplitude is different from zero only in a sector in the wake of the moving pressure point whose half angle is 19 deg 28 min; and that the ampl tude is infinite along the course behind the pressure point. In the present paper, the asymptotic expansions for the surface waves are different in different sectors radiating from the pressure point. The boundaries along the sector of half angle 19 deg 28 min are lines along which the asymptotic expansions change their character completely. The term of lowest order in the asymptotic expansion which holds for the sector coincides with Kelvin's result.

R. L. Bisplinghoff, USA

1727. **F. Ursell, On the heaving motion of a circular cylinder on the surface of a fluid,** Quart. J. Mech. appl. Math. 2, 218–231 (1949).

The author considers the wave motion generated by forced vertical oscillation of a circular cylinder with the mean position of its center at the undisturbed water surface. The analysis is based on the assumption that both waves and oscillation are of small amplitude. Expressions are obtained for the ratio of the wave amplitude at infinity to the amplitude of the oscillation and for the increase in inertia (virtual mass) caused by the motion of the water. The results of computation of these two quantities are tabulated for several values of $\sigma^2 a/g\pi$ between 0 and $^3/_2$, where σ is the frequency of oscillation, a is the radius of the cylinder, and g is the acceleration due to gravity. J. V. Wehausen, USA

1728. **Jean Legras, Aerodynamic forces on a damped oscillating airfoil** (in French), C. R. Acad. Sci. Paris 229, 699–700 (1949).

Unsteady motion of a thin airfoil of infinite span in an incompressible perfect fluid is considered. The potential and vortex-sheet strength, as well as the total circulation $\Gamma(t)$ can be calculated in two parts, of which the first neglects the effects of the vortex wake, while the second represents these effects. The relationship between $\Gamma(t)$ and the wake distribution then suffices to give an integral equation for $\Gamma(t)$. In the case of steady oscillations this is solvable in Bessel functions. [Reviewer's note: a similar method was used by von Kármán and Sears, J. aeronaut. Sci. 5, 379–390 (1938).] W. R. Sears, USA

1729. **F. Ursell, Notes on the linear theory of incompressible flow round symmetrical swept-back wings at zero,** Aero. Quart. 1, 101–122 (May 1949).

Based on the linearized theory of a perfect, incompressible fluid, a solution for the pressure distribution over a thin wing at zero angle of attack is determined from the prescribed (symmetric) thickness distribution by a direct application of Green's

theorem. Particular care is paid to the nonuniform convergence of the solution at the wing, and it is found that it is necessary to add a line integral about the point (at which the pressure is found) on the wing, depending on the sense in which the principal value of the surface integral over the wing is taken.

In the case of a swept wing formed by two semi-infinite cylinders joined at the wing center section, the author shows that the integral equation relating pressure distribution and chordwise thickness distribution can be inverted in closed form to specify the required thickness to produce a prescribed pressure. A uniqueness proof is given to demonstrate that there may be either no solution or one solution in this case, but not more than one. The application of these results to practical wing design (where the two wing halves are not semi-infinite) is discussed.

John W. Miles, USA

1730. Henry Görtler, Viscous fluids, Naturforschung und Medizin in Deutschland 1939–1946, vol. 5, pp. 33–73. Dieterich'sche Verlagsbuchhandlung, Wiesbaden, 1948. DM10 ($2.40).

This article is intended to give a summarized account of the contributions made in Germany during the indicated period on all phases of the flow of incompressible viscous fluids. The author classifies the material into eight groups, in each of which important works are enumerated and essential ideas briefly stated. The topics are: (1) Equations of motion, (2) Solutions of the general Navier-Stokes equations, (3) Surface tension, capillary waves, (4) Flows at low Reynolds numbers, (5) Foundation of Prandtl's boundary-layer theory, (6) Steady laminar boundary-layer flows, (7) Nonsteady laminar boundary-layer flows, (8) Miscellaneous applications. The review includes a short bibliography.

Y. H. Kuo, USA

1731. E. Abody-Anderlik, Friction in variable density fluid, Proc. seventh int. Congr. appl. Mech. 2, part 1, 177–186 (1948). See Rev. 3, **329.**

1732. M. S. Telang, On the mechanism of viscosity of liquids, J. chem. Phys. 17, 536–539 (June 1949).

A new viscosity-temperature relationship for liquids is suggested, namely, that the viscosity is

$$hNV^{-2/3}b(V-b)^{-4/3}\exp(1.091N^{1/3}\gamma M^{2/3}D^{-2/3}R^{-1}T^{-1})$$

where h is Plank's constant, N Avogadro's number, V the molar volume, $b = Nv_s$ and v_s is the contribution of a single molecule to the volume of the unexpanded solid, M the molecular weight, D the density of the liquid, γ the surface tension, T the absolute temperature, and R the gas constant per mole. The derivation is based on the theory of viscosity due to Eyring and others, but with the hypothesis that "activation energy" is equivalent to the free energy of formation of a surface. The proposed equation is free from any arbitrary or empirical elements, and since, in particular, it does not involve any quantities derived from viscosity measurements, it makes a direct calculation of viscosity possible. The author indicates five ways in which the validity of the equation could be studied.

L. M. Milne-Thomson, England

1733. W. R. Dean and P. E. Montagnon, On the steady motion of viscous liquid in a corner, Proc. Camb. phil. Soc. 45, 485–487 (July 1949).

The steady, two-dimensional flow of an incompressible viscous fluid in a sharp corner formed by the rigid, straight boundaries $\theta = 0, \alpha$, where r, θ are plane polar coordinates, is considered. When the flow is sufficiently slow so that inertia effects may be neglected, the stream-function ψ satisfies the biharmonic equation $(\partial^2/\partial x^2 + \partial^2/\partial y^2)^2\psi = 0$, with the boundary conditions $\psi = \partial\psi/\partial\theta = 0$ $(\theta = 0, \alpha)$.

It is shown that there are solutions of the form $\psi = r^m f(\theta)$ where m, a function of α, is real for α between 360 and about 146 deg, and complex for smaller angles. The values of m increase from $m = 1.50$ when $\alpha = 360$ deg to $m = 2.76$ when $\alpha = 146$ deg. The real part of m then increases from the latter value to 3.74 when $\alpha = 90$ deg. The variations of m, and of its real part when it is complex, are tabulated and graphed against α. These trends indicate that the larger the angle of the corner the more rapidly does the fluid velocity increase with distance from the corner.

It is also shown that, even if the motion is not slow, the foregoing solution is still valid at sufficiently small distances from the corner.

Louis Landweber, USA

1734. Sabri Ergun and A. A. Orning, Fluid flow through randomly packed columns and fluidized beds, Indust. Engng. Chem. 41, 1179–1184 (June 1949).

The flow characteristics in a packed column are investigated for the case of gas introduction at the bottom. Three separate and distinct flow regimes are considered: the fixed bed, the expanded bed, and the fluidized bed. New and previously published experimental data for the fixed and expanded beds are analyzed on the basis of the pressure gradient for flow through granular material being due to viscosity effects and expansion and contraction losses. The fixed-bed regime exists for flow rates from zero to that where the pressure difference just balances the buoyant weight of the solids per unit volume, i.e., no change in voids volume. For this case, the ratio of pressure gradient to unobstructed mean fluid velocity is a linear function of the mass rate of flow, the constants in the relationship being functions of the viscosity, particle specific surface, and voids volume. Throughout the expanded-bed regime, the expansion is homogeneous, and the general fixed-bed equation applies. Determination of the fractional expansion of the bed follows from the equation. The fluidized bed exists for flow rates greater than those required to attain the loosest stable configuration of the solids. No detailed experimental data are presented for the fluidized-bed regime (two-phase fluidization).

Richard G. Folsom, USA

1735. John Happel, Pressure drop due to vapor flow through moving beds, Indust. Engng. Chem. 41, 1161–1174 (June 1949).

The author reports some 200 tests on the concurrent and countercurrent flow of air through porous beds moving downward in 2, 4, and 8-in. pipe. The average bed-particle diameter varied between 0.015 and 0.28 in., while the fractional voids volume ϵ was between 0.38 and 0.49. Reference of the air-flow rate to the bed, rather than the pipe, properly accounted for the flow direction. However, the motion of the bed through the pipe resulted in a density which was closely approximated as a loose apparent density obtained by slowly inverting a sample in a container several times.

A plotting of the data on a friction-factor vs. Reynolds-number plot resulted in a $\pm 25\%$ correlation, with some differences from the uniform sphere fixed-bed studies of Chilton and Colburn [Trans. Amer. Inst. chem. Engrs. 26, p. 178 (1931)]. Based on an analysis of the forces on individual particles, an approximate method for modifying the friction factor and Reynolds number is developed. Correlation of the data with the aid of the modified friction factor $f/(1-\epsilon)^3$ and the modified Reynolds number $R(1-\epsilon)$ is of the order of $\pm 12\%$. This correlation is also given in equation form for the viscous, transition, and turbulent-flow regions. The test range was for modified Reynolds numbers between 0.24 and 1000. The only limit on the pressure gradient

is that with countercurrent flow the gradient be less than that which will cause boiling of the bed material.

Comparison of the regular friction factor at several Reynolds numbers, as computed by various relations given in the literature, indicated that most of the predicted pressure drops are lower than those found experimentally due to excessively large correction factors for changes in fractional voids volume. Close correlation is noted between the present formulation and that of Bakhmeteff and Feodoroff (J. appl. Mech., 1937, 1938; Trans. Amer. Geo. Union, 1943). The author reports that the correlation has been found to agree with pressure-loss measurements in commercial moving-bed catalytic cracking plants, at least within the reliability of obtaining accurate large-scale data. The paper is supplemented by a brief but thorough review of the literature based on a 56-item bibliography. J. M. Robertson, USA

Compressible Flow, Gas Dynamics

(See also Revs. 1763, 1768, 1772, 1778, 1779, 1807, 1808)

1736. **T. Y. Thomas, The fundamental hydrodynamical equations and shock conditions for gases,** Math. Mag. 22, 169–189 (1949).

In this expository paper the author develops in a clear and rapidly paced fashion the fundamental mathematical apparatus appropriate to the science of gas dynamics, and formulates side by side with these the physical principles on which this science rests. The physical laws are translated into integral form in the first instance. Time differentiation of integrals over moving volumes is explored, and the laws are then finally cast in their differential form. The distortion and viscosity tensors are introduced and related, leading to the Navier-Stokes equations. Also a brief discussion of the thermodynamics of ideal gases allows the formulation of the heat equation. Finally, the mathematical machinery previously introduced is used to derive the shock conditions for ideal gases. The paper is aimed at students possessing some mathematical and general scientific maturity. A familiarity with Cartesian tensors is assumed, for example, on the mathematical side, and on the physical side the reader is expected to be acquainted with the reduction of the stress-strain relations to their canonical form. D. P. Ling, USA

1737. **H. Poritsky, Polygonal approximation method in the hodograph plane,** J. appl. Mech. 16, 123–133 (June 1949).

In two-dimensional compressible flow, the differential equation for the stream function in the hodograph plane can be integrated approximately by the Chaplygin or the Kármán-Tsien methods, both of which depend on using a linear relation between p and $1/\rho$ in place of the adiabatic p, ρ relation. An extension of these methods, which is based on approximating the curve for the equation of state in the $(p, 1/\rho)$-plane by means of several straight line segments instead of only one, was first given by the author in Proc. sixth int. Congr. appl. Mech. (1946). In the present paper, this latter method is reviewed, further extended, and illustrated by examples. J. S. Isenberg, USA

1738. **W. P. Robbertse and J. M. Burgers, Solutions of the equations for the nonuniform propagation of a very strong shock wave,** Proc. Kon. Ned. Akad. Wet. 52: 958–965; 1067–1074 (1949).

A solution in finite terms is given for the propagation of a shock wave with increasing velocity through a gas with decreasing density. The method is based on ideas developed in earlier papers by the authors [W. P. Robbertse, thesis, Delft, 1948, Rev. 3, 742; J. M. Burgers, title source, 50, p. 262 (1947)] with the improvement of replacing a rather complicated boundary condition at the shock wave by a simpler one. The problem is solved explicitly for the case where the velocity of propagation of the shock wave greatly surpasses the flow velocities and the velocity of sound in the gas in its original condition. A specific-heat ratio of 5/3 is also assumed.

A comparison is given between the present solution and the Robbertse thesis solutions for the case in which gravity is neglected. In the thesis, since density was assumed constant, the chosen form of the solution could not be made to satisfy exactly the equation of motion and the boundary condition. Instead two alternatives were considered: the solution was made to fit exactly the equation of motion, satisfying the boundary condition at the shock wave in two points only; or else, it was made to fit exactly the boundary condition, and satisfied the equation of motion for a single layer of the gas. The three solutions agree very well for the lower range of values of velocity of the gas. A simple numerical example is also presented which indicates possible application of the solution to problems in astrophysics. Lester L. Cronvich, USA

1739. **Kl. Oswatitsch and W. Rothstein, Flow pattern in a converging-diverging nozzle,** Nat. adv. Comm. Aero. tech. Memo. no. 1215, 42 pp. (Mar. 1949); transl. from Jahrbuch 1942, Luftfahrtforschung.

In this report the authors describe a method of determining the characteristics of a steady, two-dimensional or axially symmetric, isentropic nozzle flow. In the two-dimensional case the shape of the nozzle is given as $f(x)$, where x is the coordinate along the center line of symmetry. The origin is taken so that $f(0)$ is a minimum. The usual velocity components u and v are expressed as even- and odd-powered polynomials in y with coefficients as undetermined functions of x (y^4 is the highest term considered). The coefficients are determined by the condition of irrotationality, by the Eulerian equations of motion, and by the requirement that $f(x)$ be a streamline. It is found that these conditions permit the coefficients to be determined as functions of $f(x)$ and of the unknown function $u_0(x)$, where u_0 is the velocity along the axis of the nozzle. The requirement of continuity of mass is used to complete the solution by establishing a differential equation for u_0. In order to simplify the calculations, several methods of finding approximations to the coefficients and of applying iteration are presented. The same considerations are applied to the axially symmetric case where the authors consider the first approximations cited above to be sufficient.

The methods are applied to the two-dimensional source of a compressible fluid, to an axially symmetric nozzle for which the experimentally determined velocity distribution is available, and to hyperbolic two-dimensional and axially symmetric nozzles. Paul A. Libby, USA

1740. **P. Chiarulli, Stability of two-dimensional velocity distributions of the half-jet type,** Hdqtrs. Air Mat. Comm. Dayton Tech. Rep. no. F-TS-1228-IA, 50 pp. (June 1949).

The theory has long been held that laminar flow may be subject to instability under certain conditions, and that this accounts for the transition to turbulent flow at a certain Reynolds number. It was shown by Rayleigh that for nonviscous fluids such instability can occur only if the curve of velocity distribution has a point of inflection. This paper derives a general expression for the velocity distribution resulting from the laminar mixing of of any two uniform flows with velocities of different magnitude but the same direction (distributions without solid boundaries), and makes a study of the stability characteristics of the half-jet (the special case of such a mixing when one of the flows is stationary). A similar study is made of another velocity distribu-

tion of the same general type. Viscosity effects are taken into account. Previous stability studies have been made for both compressible and incompressible fluids with various velocity distributions, but the solution here given differs in that boundary conditions are imposed at an infinite distance from the origin (the velocity approaching a constant value at infinity), whereas previously the boundary conditions have been imposed at a finite distance. C. W. Smith, USA

1741. A. D. Young, Note on the limits of the local Mach number on an aerofoil in subsonic flow, Proc. seventh int. Congr. appl. Mech. 2, part 2, 425–436 (1948).

See Rev. 3, **750**.

1742. Adolf Busemann, The drag problem at high supersonic speeds, J. aero. Sci. 16, 337–344 (June 1949).

The title should read ". . . . subsonic speeds."

The paper visualizes an arbitrary, nonlifting, two-dimensional body moving in a nonviscous fluid at a subsonic speed sufficiently high so that a local region of supersonic flow is present. The problem considered is to determine the conditions for which shock waves and wave drag will first appear. A rigorous discussion of this problem is presented without the use of mathematics. The physical characteristics of the local supersonic region are described in detail, and the question is raised as to whether a flow disturbance originating at the body surface will be amplified in this region. In order to answer this question, the effect of superimposing a disturbance on the steady supersonic flow in a channel is considered. It is shown that the disturbance tends to be amplified in decelerating flow and reduced in accelerating flow. Thus, in the local supersonic region of the body, shock waves tend to form in the decelerating flow near the downstream end of the region. This result is in agreement with experiment.

The author points to the fact that this tendency of disturbances to amplify in decelerated supersonic or mixed flows casts doubt on the existence of shock-free potential flow with mixed subsonic-supersonic velocity patterns. The relatively few instances of genuine potential flows with mixed patterns that have been discovered are considered rare exceptions. In the usual case the first appearance of shocks and wave drag coincides with the attainment of local sonic velocity. The author's opinion is that the most important problem in mixed flows is to determine the wave drag for given body shapes with "the usual order of workshop accuracy," rather than to seek for the rare exceptions which (theoretically) do not develop shocks immediately upon the occurrence of local sonic velocity.

Reasonable solutions to the practical problem of drag evaluation have already been obtained in a few cases. The author cites as an example the recent work of G. Guderley, who obtained the flow pattern and drag of an airfoil at Mach number 1.0. An especially significant result of Guderley's work is the occurrence of finite drag for the optimum profile shape considered. This result proves the definite impossibility of approaching Mach number 1.0 with any genuine potential flow having zero drag.

Aerodynamicists will be grateful for the author's enlightening, nonmathematical discussion of a complex and important problem. They will also heartily endorse his view that future efforts in this field should concentrate on the problem of evaluating the transonic drag for specified body shapes. John V. Becker, USA

1743. Theodore R. Goodman, The lift distribution on conical and non-conical flow regions of thin finite wings in a supersonic stream, J. aero. Sci. 16, 365–374 (June 1949).

The linearized theory is used to find the lift distribution over thin finite wings in a supersonic flow. The procedure is to start with the results which are applicable to wings of infinite extent, and then modify them for the case of finite wings by superimposing additional solutions which cancel the surplus lift in the regions where the infinite wing is removed. In the present paper particular solutions representing supersonic doublets are used to remove the excess lift. The results are valid for nonconical flows, as well as for cases in which the various regions of excess lift lie in the zones of influence of each other. (Wings with two subsonic trailing edges meeting at a point, however, must be excluded.) Several examples are then given to illustrate the method. Hideo Yoshihara, USA

1744. John R. Spreiter, Aerodynamic prope ties of cruciform-wing and body combinations at subsonic, transonic, and supersonic speeds, Nat. adv. Comm. Aero. tech. Note no. 1897, 32 pp. (June 1949).

This report is a continuation of no. 1662 (1948), same source (Rev. 1, **1386**) by the same author. The computation method is based on assumptions similar to those of Munk's airship theory and R. T. Jones' low-aspect-ratio pointed-wing theory. That means that in the equation

$$(1 - M_0^2)\, \varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0$$

the second and the third terms are more important than the first one, so that the problem is governed by

$$\varphi_{yy} + \varphi_{zz} = 0$$

which is Laplace's equation in the transverse plane yz. Therefore, in a first approximation, the slender bodies and low-aspect-ratio wings are independent of the Mach number. The results are valuable at subsonic, transonic, and supersonic speeds, provided that the entire wing-body combination lies near the axis of the Mach cone. After having treated ordinary thin wing forms with bodies of circular cross sections in the previous report, the author treats in this report cruciform wing and body combinations, and studies also the effect of yaw. The flow in the transverse planes is computed with the help of conformal mapping. Symmetry considerations lead to some general conclusions regarding the influence of angle of attack and angle of yaw, and check thus the results of the detailed computations. Lift, side force, pitching, and yawing moment are computed for several special forms of wing-body combinations. I. Flügge-Lotz, USA

1745. John W. Miles, The aerodynamic forces on an oscillating flap at supersonic speeds, J. aero. Sci. 15, 565–568 (Sept. 1948).

The work of an earlier paper [J. aero. Sci. 14, 351–358 (June 1947); Rev. 1, **156**], giving the pressure on an oscillating flat-plate airfoil in a supersonic stream, has been extended in this paper to include a flap. Several errors are present in equations and notation, which are subsequently corrected (same source, 16, 442–443, July 1949). The real and imaginary parts of the complex air-force coefficients are presented as families of curves with Mach number as parameter covering the range M = 1.2, 1.4, 1.6, 2.0. Numerical results are claimed to be inaccurate, the latter communication pointing out Wright Field reports as giving more accurate results. Walter W. Soroka, USA

1746. J. H. Hunter-Tod, The aerodynamic derivatives with respect to rate of yaw for a delta wing with small dihedral at supersonic speeds, Coll. Aero. Cranfield Rep. no. 28, 20 pp. (Mar. 1949).

The linearized supersonic flow theory is used to obtain the stability derivatives with respect to rate of yaw of a delta wing having small dihedral. The results show that the derivatives de-

crease numerically with increasing Mach number. The destabilizing action of the leading-edge suction force, which is present when the leading edge lies behind the apex Mach cone, is found to be of comparable magnitude to the destabilizing effect of the induced excess-pressure distribution.

John V. Becker, USA

1747. **A. Robinson, On some problems of unsteady supersonic aerofoil theory,** Proc. seventh int. Congr. appl. Mech. 2, part II, 500–514 (1948).

See Rev. 2, **1425**.

1748. **F. Cap, A coupling of flow and combustion processes** (in German), Öst. Ingen.-Arch. 3, no. 2, 97–106 (1949).

The author has made an attempt to modify the principal equations of aerodynamics in such a way as to account for the accompanying combustion. By using the Charbonnier-Schmitz-Vieille law of combustion, the equation for the characteristics of the potential equation is obtained in the form

$$du/dt \pm 2\,(u-1)^{-1}\,da/dt = q^{*}du/dt - \bar{q}(u-u^{*}) \pm q'a.$$

Under certain assumptions a graphical procedure based on this equation leads to the solution of nonsteady one-dimensional aerodynamical problems of flow accompanied by combustion (rockets, cannons, ramjets). [Cf. F. Cap, Acta Phys. Austriaca 1, 89–97 (1947)].

E. Leimanis, Canada

1749. **Numa Manson, Henri Guenoche, and Bernard Sale, On the calculation of the speed of deflagration of gases** (in French), C. R. Acad. Sci. Paris 228, 1637–1639 (May 23, 1949).

The velocity of the flame relative to the unburned gas in a spherical bomb is computed for the four mixtures: $C_6H_6 + 7.5\ O_2 + 12.9\ N_2$, $C_7H_{16} + 11\ O_2 + 19\ N_2$, $C_8H_{18} + 12.5\ O_2 + 21.5\ N_2$, and $CO + 0.5\ O_2 + 0.0415\ H_2O$. The calculations assume the mechanism of flame propagation to be one of diffusion of hydrogen atoms into the unburned gases. Isentropic compression and equality of pressure between the burned and unburned gases was assumed. Comparisons are made between the calculated values and the experimental values presented in Nat. adv. Comm. Aero. Rep. no. 682 (1940); agreement between observed and calculated values is fair.

J. Howard Childs, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 1730, 1740)

1750. **Maurice Roy, Laminar boundary layer on a helicoid in uniform rotation** (in French), C. R. Acad. Sci. Paris 228, 1994–1996 (June 1949).

This is a brief presentation of a form of approximate boundary-layer equations for the flow near the surface of a semi-infinite helicoidal surface whose axial translational and rotational speeds are related so as to give zero "angle of attack" as it moves through an infinite fluid. The term "semi-infinite" means that in cylindrical coordinates (x, r, θ) the surface is in the space $x \geqq 0$ $r \geqq r_0$. Translation (of velocity V) is in the negative x-direction; rotation is about the x-axis.

Appropriate orthogonal coordinates are: s along the helicoidal plane, n along the perpendicular helix, and radial (r). The kinematic viscosity is assumed small $(Vr_0/\nu \gg 1)$, and presumably the boundary-layer thickness is small compared with the pitch of the helix, since this flow is regarded as a generalization of the flat-plate boundary layer.

Although the resulting boundary-layer equations are quite formidable and have not been solved, some interesting conclusions have been drawn: (a) The radial velocity v_r in the boundary layer is of the same order as the tangential helical velocity v_s. (b) There appears to be no simple geometrical similarity law for either v_s or v_r.

Stanley Corrsin, USA

1751. **S. Chandrasekhar, The theory of statistica and isotropic turbulence. On the decay of isotropic turbulence,** Phys. Rev.: 75, no. 5, 896–897; 76, no. 1, p. 158 (1949).

The author obtains an explicit formula for the spectrum of turbulence at high frequencies from an equation of Heisenberg, who gave only an interpolation formula. The present formula is $F(k) = F(k_0)\,(k_0/k)^{5/3}\,[1 + (k/k_s)^4]^{-4/3}$, where $F(k)$ is the spectrum, and k_0 and k_s are two typical wave numbers $(k_0/k_s \ll 1)$. [Reviewer's remark: Essentially the same result is said to have been obtained independently by J. Bass.]

This note shows that the author's analysis of the spectrum of turbulence agrees with Heisenberg's result for large frequencies. This clarifies an apparent contradiction announced in the note of Rev. 3, **1348**.

C. C. Lin, USA

1752. **W. Heisenberg, On the statistical theory of turbulence** (in German), Z. Physik 124, 628–657 (1948).

This paper develops the statistical theory of isotropic turbulence from the point of view of spectrum analysis. The author develops the relation between the spectra obtained from a one-dimensional Fourier analysis and from a three-dimensional analysis. An assumption is introduced regarding the transfer mechanism, leading to a definite law of spectrum for the range of frequencies chiefly responsible for dissipation. [In this paper, the author uses an interpolation formula; but the exact law has been obtained later by Chandrasekhar, Phys. Rev. 75, 896–897 (1949); see preceding review.] The mean pressure fluctuation and the velocity correlation function are calculated, and the constant of proportionality appearing in the transfer function is estimated. [It should be noted that the above two papers were submitted in 1946. In the meantime, developments based on these ideas have already been published.]

C. C. Lin, USA

1753. **R. Michel and M. Ménard, Transition and turbulence of the boundary layer** (in French), Off. nat. Étud. Rech. aéro. Rep. no. 33, 23 pp. (1949).

Measurements are described of the mean velocity (u) and the rms value of the longitudinal component of the turbulent velocity (u') at different heights (y) in the boundary layer of an airfoil and at various distances (x) between the leading and trailing edges. Both velocities were measured by means of a hot-wire anemometer coupled to the input of a commercial amplifier, uncompensated for the heat inertia of the wire. The measurements were made to determine the position and extent of the transition region.

The measurements of u' at constant y and varying x showed that between the leading edge and the end of the laminar layer (denoted by X_1) the intensity of the turbulence was sensibly uniform. The value of u' then increased rapidly to a maximum at the point X_2, and after falling again became sensibly uniform at a point X_3. The authors refer to X_1 as the transition point, and to X_3 as the point at which a fully developed turbulent boundary layer is established.

Comparison with other experimental methods of indicating transition showed that whereas chemical methods indicated point X_2 as the transition point, Jones' method, based on the distribution of u against x for constant y, determined points A and B, comparable with X_1 and X_3 respectively, but which differed with the height of measurement from the surface. In the latter case agreement with the turbulence measurements was obtained,

when the positions of the points A and B at various values of y were extrapolated to the values at $y = 0$.

G. M. Lilley, England

1754. **M. K. Baranaev, E. N. Teverovskiĭ, and E. L. Tregubova, On the magnitude of the minimal pulsations in a turbulent flow** (in Russian), Doklady Akad. Nauk SSSR 66, no. 5, 821–824 (June 1949).

According to the usual conception of dissipation of the energy of turbulent flow into heat, the macroscopic pulsations, defined by the dimensions of the turbulent flow, dissipate to pulsations of smaller dimensions, and finally the pulsations of minimal scale dissipate directly into heat. The scale of these minimal pulsations λ_0 is determined by the condition that the Reynolds number, corresponding to their scale, $Re_{\lambda_0} \rightarrow 1$. If the turbulent flow contains some droplets of another insoluble liquid, it is believed that the minimal size of such droplets is identical with the size of minimal pulsations. The surface tension can be expressed by means of the Weber number, which too should approach 1. From these two conditions the magnitude of minimal droplets can be determined from measured Re and the known surface tension. In a special experiment the size of minimal droplets for two special liquids, projected into the turbulent flow of water, was measured by a microscope and found 10^2 smaller than the theoretical values. This disagreement is, among others, explained by the presence of local velocity gradients much larger than the mean value of maximum-velocity gradient computed from the dissipation of the energy of turbulent flow.

Z. Sekera, USA

1755. **A. N. Kolmogorov, On atomization of droplets in a turbulent flow** (in Russian), Doklady Akad. Nauk SSSR 66, no. 5, 825–828 (June 1949).

This note comments on a paper by Baranaev, Teverovskiĭ and Tregubova (see preceding review), describing a method for measuring the smallest pulsations in a turbulent flow of water by measuring the average size of the droplets of another insoluble liquid introduced into the flow. It is pointed out first that the authors have not correctly interpreted the microscale λ_0 occurring in Kolmogorov's theory. An analysis is then given of this type of experiment using dimensional arguments somewhat similar in nature to those used previously by the author in his turbulence theory. It does not seem feasible to describe the results briefly. Finally, some difficulties in the interpretation of such experiments are mentioned, and some desirable future developments of the method are outlined.

J. V. Wehausen, USA

1756. **J. O. Hinze and B. G. van der Hegge Zijnen, Transfer of heat and matter in the turbulent mixing zone of an axially symmetric jet,** Appl. sci. Res. Sec. A A1, no. 5–6, 435–461 (1949) = Proc. seventh. int. Congr. appl. Mech. 2, part 1, 286–299(1948).

This paper is a report of some very careful measurements of mean velocity, temperature and concentration in round fully developed turbulent jets at low subsonic speeds, with an essentially constant density system. As pointed out by the authors, it has long been known that heat diffuses laterally more rapidly than momentum in such a turbulent shear flow. Their principal new result is the discovery that matter, a scalar "property" like heat, diffuses at the same rate as heat, with identical results for several different gases used as "tracers" in an airjet.

The reviewer would like to call attention to an independent study by W. R. Keagy and A. E. Weller of the Battelle Memorial Institute (Proc. of the Heat Transfer and Fluid Mechanics Inst., Berkeley, California, 1949, ASME) in which the width ratio of concentration to velocity distribution was different for different jet gases. However, the measurements at Battelle were done in jets of 100% initial gas concentration rather than 1%, as in the present work.

Comparing their mean velocity distributions (radially) with the shapes predicted by the three most popular semiempirical "theories" for turbulent shear flow, they find, in agreement with the reviewer, that the constant exchange-coefficient "theory" gives better agreement over the fully turbulent jet core than do the momentum transfer and modified vorticity transfer "theories." They find, however, that none of these analyses gives an adequate agreement with the experimental distributions of concentration and of temperature.

The total-head tube readings were corrected approximately for the effect of turbulence. The authors remark that the (chromel-alumel) thermocouple readings should have been corrected for the effect of velocity and temperature fluctuations, and that the sampling-probe operation is unaffected by fluctuations. The reviewer would like to remark that, if inertia effects be neglected as in the pitot tube corrections, then (a) the (essentially linear) thermocouple reads average value correctly in the presence of fluctuations, while (b) the sampling process, which is nonlinear in velocity, must, in principle at least, read in error.

Stanley Corrsin, USA

Aerodynamics of Flight; Wind Forces

(See also Revs. 1729, 1744, 1746, 1763, 1766)

1757. **John W. Miles, The rolling moment due to sideslip for a swept wing,** J. aero. Sci. 15, 418–424 (July 1948).

1758. **Robert T. Jones and Leonard Sternfield, A method for predicting the stability in roll of automatically controlled aircraft based on the experimental determination of the characteristics of an automatic pilot,** Nat. adv. Comm. Aero. tech. Note no. 1901, 14 pp. (June 1949).

A closed-loop analysis is made of open-loop data, combining calculated airplane frequency responses with experimental autopilot frequency responses. The airplane response in roll is computed from a simplified one-degree-of-freedom expression, which seems valid at frequencies concerned with autopilot stability. The nonlinearities in the autopilot (dead spots and friction) are approximated experimentally by a linear system for each amplitude of input. The results for the autopilots tested showed less lag for larger amplitudes.

The closed-loop stability is determined qualitatively for several amplitudes by using, in effect, the Nyquist criterion. These results determine the amplitude and frequency at which constant amplitude hunting occurs.

A qualitative method of calculating autopilot requirements to damp disturbances at various exponential rates is presented, although it is pointed out that this method does not apply to nonlinear systems with dead spots. The sinusoidal frequency response with the Nyquist diagram accomplishes the same thing with a usable experimental technique.

Graham F. Campbell, USA

1759. **A. D. Wood, Flight tests on undercarriage loads for a single-engined aircraft equipped with skis,** Nat. Res. Counc. Canada Mech. Engng. Rep. no. MR-7, 79 pp. (June 1949).

Results are presented of a series of tests to determine the maximum loads experienced by a ski undercarriage while landing and during other ground maneuvers. Tests cover two types of skis and a wide range of snow conditions. John R. Spreiter, USA

1760. **A. A. Nikolsky and Edward Seckel, An analytical study of the steady vertical descent in autorotation of single-rotor helicopters,** Nat. adv. Comm. Aero. tech. Note: no. 1906, 28 pp.; no. 1907, 29 pp. (June 1949).

The first report is based initially on the well-known theories of Locke and Glauert on autorotative descent in the windmill brake and vortex ring states. To simplify the analytical treatment, however, the relationship between the nondimensional coefficients f and F is written in the form of $1/f = 2 \pm K/F$. The relationship between f and F for the whole rotor is assumed to apply for any blade section. These relationships are then introduced into the customary equations for T and Q. $Q = 0$ since there is autorotation. The ratio $V/\Omega R$ for $Q = 0$ is found for variable induced velocity and for constant induced velocity over the disk. Stability of autorotation is then investigated for a single blade element. Calculations are then made for a typical helicopter on the assumption of constant induced velocity and disregarding possible stall along the blades, and with certain other limitations the following conclusions are (tentatively) advanced:

1. Rate of descent and rotor speed are not critically affected by different assumptions for rotor-thrust coefficient based on descending velocity f against rotor-thrust coefficient based on resultant velocity F in the range of conditions encountered in steady autorotative descent.

2. For the computation of rate of descent and rotor speed, constant induced-velocity theory may be used at low incidence where stalling may be neglected. At high incidence, blade stalling must be accounted for in order to obtain even qualitative agreement between theory and practice. For quantitative agreement in this case, it would probably be necessary to use a variable induced-velocity theory.

3. At high values of incidence, although the autorotation may be stable for infinitesimal disturbances, a finite disturbance such as an upgust might well stall enough of the blades to put the rotor in an unstable regime where it would cease autorotating. There is little danger of this, at least for aerodynamically clean blades, at low incidence.

4. For the sample design studied, the constant induced-velocity theory, accounting for blade stalling, indicates a critical value of blade incidence of about 8.8 deg, above which steady autorotation would not be possible. Alexander Klemin, USA

The results of the second report indicate that the effect of blade flapping is negligible as far as the establishment of steady autorotation is concerned. From the standpoint of avoiding blade stalling during the transition, it is desirable to reduce pitch rapidly and to have a large blade moment of inertia. The results calculated for a typical helicopter show that the average time to effect the transition from hovering to steady autorotation is about 6 sec, and the corresponding average altitude lost in the transition is about 120 ft. S. W. Yuan, USA

Aeroelasticity (Flutter, Divergence, etc.)

1761. **Donald S. Woolston and Harry L. Runyan, Appraisal of method of flutter analysis based on chosen modes by comparison with experiment for cases of large mass coupling,** Nat. adv. Comm. Aero. tech. Note no. 1902, 16 pp. (June 1949).

The present report gives some comparisons of experiments and simplified numerical analysis of Rayleigh energy method with a few assumed uncoupled modes. The structural model under flutter study is a straight uniform cantilever wing of high aspect ratio loaded with a single concentrated weight at various locations. The results of comparisons are:

1. Good agreement in the case of the wing alone, or in the case of the wing carrying a concentrated weight near the elastic axis.

2. The simple analysis is conservative or unconservative according to the location of the weight behind or forward of the elastic axis, respectively. In order to improve the agreement, some more modes such as second modes in bending and torsion are necessary to be considered, because the mass coupling increases with the weight located more ahead of or behind the elastic axis. Coupling modes and damping are not considered. Chieh-Chien Chang, USA

1762. **Manfred Rauscher, Station functions and air density variations in flutter analysis,** J. aero. Sci. 16, 345–353 (June 1949).

In problems of dynamics it is frequently necessary to replace distributed masses by a finite number of concentrated masses in order to render the analysis possible. The accuracy resulting from this simplification depends on the number of mass stations chosen which correspond to the degree of the characteristic equation and hence to the labor involved.

In the method of station functions, continuous-function features are incorporated in the finite station scheme. Continuous inertia loads are referred to the finite number of stations by the curve

$$f(x) = y_1f_1(x) + y_2f_2(x) + \ldots + y_nf_n(x)$$

where $y_1, y_2, \ldots, y_n$ are the deflections at $x_1, x_2, \ldots, x_n$, and $f_1(x)$ the continuous interpolation functions chosen to give the proper deflection at the stations. This addition of the continuous-function feature leads to good accuracies with only a few stations. Given as an example is the comparison of accuracies for the two-station reduction of a cantilever beam for which the concentrated mass method leads to an error of 10 and 26% for the first and second natural frequencies. With the same two stations this error is reduced to 1 and 5% by the linear interpolation function, and 0.2 and 1.2% by the parabolic interpolation function.

Station functions can be adapted for the method of influence coefficients. Suitable interpolation functions can be developed for beams with variable elastic as well as inertial properties. Problems on torsion and flexure-torsion can be treated by this method. It is possible to define surfaces by station functions, thus making possible the vibration analysis of flat sheets.

The paper describes the application of station functions to problems in aircraft flutter. The incorporation of the aerodynamic-span effect to established methods of discrete span stations is mentioned. A treatment of the flutter problem using the air density as a variable is given, thus making available flutter performance at various altitudes.

The paper serves mainly as a summary of a more complete account given in a Navy Bureau of Aeronautics report. W. T. Thomson, USA

1763. **Warren A. Tucker and Robert L. Nelson, The effect of torsional flexibility on the rolling characteristics at supersonic speeds of tapered unswept wings,** Nat. adv. Comm. Aero. tech. Note no. 1890, 69 pp. (June 1949).

As a continuation of the author's early work [same source, no. 1769 (1948)] this paper shows the effect of torsional flexibility on the rolling characteristics of tapered unswept wings. The ailerons are of constant chord length and extend inboard from the wing tip over a part of the wing span. The assumptions are: (a) elastic behavior of the wing under torsion; (b) the twisting moment on the wing results solely from the aerodynamic loads of the deflected ailerons; (c) the twisting moments on the wing resulting from rolling and twisting are neglected (of course, these additional contributions can be calculated and added under the principle of

superposition); (d) the aileron does not twist with respect to the wing.

In the analysis, first the pressure distributions on the aileron are treated with linearized supersonic wing theory. Next, the method of finding the twist along the span is outlined. Last, the numerical calculations are shown in graphs for different geometrical parameters, and a computation form is provided to be used with the graphs. Chieh-Chien Chang, USA

1764. Alexander Mendelson, Effect of centrifugal force on flutter of uniform cantilever beam at subsonic speeds with application to compressor and turbine blades, Nat. adv. Comm. Aero. tech. Note no. 1893, 37 pp. (June 1949).

Calculations are presented for the incompressible flutter of axial-flow compressor and turbine blades of the uniform cantilever variety. Two types of solution are obtained: In the first, the blade is replaced by a two-dimensional (rigid wing) flutter system with the same mass characteristics as the blade, and having natural frequencies in bending and torsion which are equal to the fundamental uncoupled bending and torsion frequencies of the blade when rotating *in vacuo*. In the second solution, the wing is treated as a rod with bending and torsional stiffness, and the partial differential equations governing the flutter vibrations of the rod are solved; the aerodynamic treatment is by strip theory based on the two-dimensional coefficients.

Comparison of solutions obtained for two-blade systems by the two methods show excellent agreement, thus suggesting that the two-dimensional simplified system is adequate for practical work. Calculations on this basis are presented for a wide variety of blade systems.

Apparently no consideration is given to the spanwise variation in reduced frequency as a result of the velocity variation introduced by the blade rotation. This factor is not important in blades of small aspect ratio, but in these cases the use of strip aerodynamic theory cannot be justified. Martin Goland, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 1634, 1706, 1760, 1764, 1776, 1815)

1765. E. Eckert and K. von Vietinghoff-Scheel, Experimental study of flow past turbine blades, Nat. adv. Comm. Aero. tech. Memo. no. 1209, 29 pp. (June 1949).

To provide basic data for the design of turbine blading, the flow through a blade grid of highly curved profiles was analyzed with the Mach-Zehnder interferometer. From the records obtained with this method, the density field and the pressure distribution in the vicinity of the grid were obtained. Since the boundary layer of the flow was visible, separation phenomena could be studied. The experiments were carried out at Reynolds numbers usual for actual turbines. The model blades were relatively large, so that the speed had to be low. At too low speeds the visibility of the boundary layer had to be increased by means of electrical heating of one of the blades.

The flow was examined at different spacing ratios and flow angles and one form of profile. The results are given in a number of interference photographs showing favorable combinations of spacing ratios and flow angles and cases in which separation occurs. J. G. Slotboom, Holland

1766. Paul J. Carpenter and Herbert E. Peitzer, Response of a helicopter rotor to oscillatory pitch and throttle movements, Nat. adv. Comm. Aero. tech. Note no. 1888, 31 pp. (June 1949).

Drag-angle oscillation of a helicopter rotor was measured with the Langley helicopter apparatus, and both the natural frequencies and the damping required to prevent excessive drag-angle movement were determined. Methods of calculating both the symmetric and unsymmetric drag-angle frequencies are presented, and the experimental results show fair agreement with the predicted values. The damping calculations show that the drag-hinge damping added to the system to prevent resonance caused by main-pitch or throttle movements should be approximately 35% of the critical value. S. W. Yuan, USA

1767. Ch. Hanocq, Investigation of helical fans (in French), Hommage Faculté Sci. appl. Liége, 105–125 (1947).

This is an attempt at an elementary theory of the lift, based on the momentum theorem alone, and on some empirical results. Gino Moretti, Argentina

1768. Ambrose Ginsburg, Irving A. Johnsen, and Alfred C. Redlitz, Determination of centrifugal-compressor performance on basis of static-pressure measurements in vaneless diffuser, Nat. adv. Comm. Aero. tech. Note no. 1880, 29 pp. (June 1949).

This paper investigates the use of measured static pressure in a vaneless-diffuser passage, for determining centrifugal compressor performance. If static-pressure measurements permit a reliable prediction of the available energy in the air, the performance of impeller and diffuser can be analyzed separately at great savings in time and with simple instrumentation.

The general effects of diffuser-wall-surface friction were studied to locate the regions in the vaneless-diffuser passage where the most valid evaluation can be made by the static-pressure method. For the most accurate determination of compressor performance from measured static pressures, a wall-surface friction correction is required. The friction correction, however, is small at the diffuser entrance and in the region of low kinetic energy at the diffuser exit. The equations for consideration of the friction correction are derived, and a value for the friction factor is recommended.

The compressor ratings based on measured static pressure and on measured total pressure are compared for 3 different compressors of about the same physical dimensions. The investigation of the authors indicates that at the diffuser entrance the efficiencies determined from measured total pressures were considerably higher than those based on measured static pressures and calculated total pressures for the range of volume flow and tip speed, except at the low volume for a high tip speed of 1200 fps where the calculated total pressures were slightly higher. The performance based on calculated total pressures and measured total pressures gave nearly the same result at the diffuser exit under the entire range of speed and flow. H. E. Sheets, USA

1769. Jean Lefol and Lucien Lefol, Cavitation criteria (in French), Bull. Ass. tech. Marit. Aéro. 47, 429–453 (1948).

Former experiments by Lerbs on the cavitation properties of methodical series of 3- and 4-bladed model propellers (see e.g. van Lammeren, Troost, and Koning, ***Resistance, propulsion and steering of ships***, Haarlem, 1948) are represented by an interpolation formula with respect to the advance coefficient at which the thrust begins to decrease. Within a methodical series, this relation is a three-parameter family of curves which depends on cavitation number, blade-area ratio, and pitch ratio. Choosing as the variable the cavitation number times blade-area ratio, the relation is reduced to two parameters (as follows from thin airfoil theory). The model results, as expressed by the interpolation formula, are in agreement with full-scale trial results on destroyers, published by Dieudonné (Ass. tech. Marit. Aéro., 1947).
Louis Landweber, USA

1770. **J. D. Van Manen, Investigation of the possibility of representing correctly in a tunnel the cavitation phenomena actually occurring on propellers** (in Dutch, with English summary), Ingenieur 's-Gravenhage 61, 25–38 (June 1949).

The effects of the mixed wake due to the hull results in changes of speed and direction of flow in the propeller disk. On the other hand, cavitation-tank tests of the screw have to be made in the homogeneous flow of the cavitation tunnel. The author considers first in some detail the design of a wake-adapted screw adopting a method due to Lerbs, slightly modified to correct an error in the application of the momentum principle. He then proceeds to consider the design of a "comparison-screw" which shall show in the homogeneous flow of the tunnel the same cavitation phenomena as the propeller behind the ship. The whole matter is discussed in considerable detail, and a definite conclusion is reached as follows: In present-day tunnel trials wherein the wake-adapted screw is tested at a cavitation number deduced from the physical data of the ship, more trustworthy results are obtainable than with the comparison-screw, since the former gives a better control in the cavitation region, the tip, than the latter. This appears to be an inherent defect in comparison-screws, however designed. L. M. Milne-Thomson, England

Flow and Flight Test Techniques

(See also Revs. 1623, 1765, 1770, 1815)

⊂1771. **E. Ower, The measurement of air flow,** London, Chapman & Hall Ltd., 1949, vi + 293 pp. Cloth, 5.75 × 8.75 in., $4.20.

This is purely a scholarly treatment of the science of measuring the flow rate of air, presented at college level for engineers and scientists. While this reviewer is not familiar with the British or continental fields of literature, there is no doubt that American engineers will find the text extremely valuable as an informatory source for testing air-moving and air-consuming machinery, for heat-transfer studies and other applications requiring accuracy of a high order.

The text, to the nontechnical practical operating or design engineer, is discouragingly replete with calculus; there are too few short, working formulas. For that reason it should appeal more to physicists and research scientists.

All known methods and devices for gaging the flow-rate of an air stream are thoroughly expounded, with the limitations and disadvantages of each. A welcome innovation is the inclusion, as a final chapter, of "Examples from practice," something sadly needed in the popular American handbooks on engineering subjects.

The book is built around the subject of air flow in pipe lines and in ducts, at fairly low pressures as are found in fan and blower work, in air conditioning, etc. As such, it may be felt not suitable for high-pressure studies as encountered in compressor and liquefaction plants, although the principles laid down would apply to all pressure levels.

Those already familiar with American expressions and terminology in flow measurement will find the British usage a little hard to become accustomed to, and the going, therefore, gets a little rough in following through the mathematical derivations.

The ASME Fluid Meter Committee Reports and Power Test Codes and the ASHVE publications cover the subject equally as well as this text for perhaps 90% of the potential reader market. R. L. Galley, USA

1772. **W. J. Duncan, A simple approach to wind tunnel constriction effect,** Aircr. Engng. 21, 180–183 (June 1949).

This paper shows that an exact value for the mean interference velocity due to the pressure of a test model in a closed tunnel can be obtained from consideration of the continuity requirement. The interference velocity can be expressed in terms of the perturbation velocity due to the same model in free air. The linearized theory is applied and it is found that the compressibility effect on the interference velocity for small two- or three-dimensional bodies is given by the factor $1/\beta^3$, where $\beta^2 = 1 - M^2$, and M is the Mach number. This result is in agreement with the work of previous investigators. For the case of a wake or body having a long parallel-sided middle section the constriction effect is shown to vary in proportion to $1/\beta^2$. John V. Becker, USA

1773. **H. E. Moses, Velocity distributions on arbitrary airfoils in closed tunnels by conformal mapping,** Nat. adv. Comm. Aero. tech. Note no. 1899, 45 pp. (June 1949).

Conformal-mapping methods are applied to the calculation of wall effects for the case of incompressible flow past an arbitrary airfoil situated anywhere within an arbitrarily shaped two-dimensional tunnel. This is an extension of previous work by the same author and W. Perl [same source, no. 1642 (1948)]. The velocity distribution on the airfoil and on the channel walls is obtained.

The method is illustrated numerically by examples. The results for the case of a symmetric airfoil placed at the center of the channel are compared with corresponding results given by the first-order image theory and by the second-order image theory of Goldstein. J. S. Isenberg, USA

1774. **I. Estermann and E. D. Kane, A torsion balance for measuring forces in low density gas flows,** J. appl. Phys. 20, 608–610 (June 1949).

The authors describe a torsion balance intended to measure the force exerted upon obstacles by supersonic gas flows with static pressure so small that the molecular mean free path is large compared to the dimensions of obstacles. The torsion member used in the balance is a tungsten wire. The authors state the calibration procedure and the results obtained on a flat plate of a section of 0.019 sq in. for static pressures of the flow ranging from 10 to 80 micron, with corresponding forces of 0.1 to 0.2 mmg. Carlo Ferrari, Italy

1775. **Thomas W. Williams and James M. Benson, Preliminary investigation of the use of afterglow for visualizing low-density compressible flows,** Nat. adv. Comm. Aero. tech. Note no. 1900, 23 pp. (June 1949).

A new flow visualization technique, utilizing afterglows and applicable to low-density supersonic wind tunnels, is described. An afterglow is a "luminescence that persists in certain gases for an appreciable time after the gases are excited to states capable of emitting light." Commonly used optical methods for flow visualization in supersonic gas streams depend on the variation with density of the index of refraction of air for visible light wave lengths. As the average gas density is decreased, these methods (interferometer, schlieren, and shadowgraph) become less sensitive, and there is a lower limit of gas density for which a given system is operable. This lower limit is reached and exceeded, for example, in some proposed hypersonic wind-tunnel installations, or in equipment designed to investigate the phenomena of "superaerodynamics." The method in this paper consists of arranging an electrical discharge tube upstream from the supersonic nozzle throat, so that the gas entering the nozzle first passed through the discharge. A portion of the gas thus was excited to the afterglow state, and the glowing gas passed through the nozzle and around a model in the test section. Flow patterns were observed

in a wind tunnel operated with argon, oxygen, and air, but nitrogen gave the best results. References are quoted in which the intensity of an afterglow was observed to increase with a sudden increase in gas density, and to this phenomenon the authors attribute the variations in glow intensity apparent in their photographs.

Comparative schlieren photographs indicate that the glow method was effective in visualizing some of the features of the flow in a range of low densities where the schlieren system was not useful. In a 1.5-in. wide nozzle, the schlieren system was inoperative at a test section pressure of 3 mm Hg (density 1.9×10^{-5} slugs per cu ft, Mach number = 2.6), but satisfactory pictures were obtained under these conditions with a nitrogen afterglow. Complete details of the simple experimental arrangement are given in the paper. An indication of the brightness of the nitrogen afterglow are the exposure times of from 20 to 45 sec required for photographs with an f/1.5 lens. The authors conclude that their preliminary results appear to justify use of the afterglow method to show general features of the flow, and that additional investigations are required to determine quantitatively the useful range and the limitations of this new technique.

Additional results have been obtained with glow methods at lower gas densities using slightly different excitation arrangements [E. D. Kane, Preliminary report on an electrical discharge method for flow visualization in a low density supersonic wind tunnel, Univ. Calif. Eng. Project Report HE-150-51 (Feb. 1949); H. L. Dryden, The aeronautical research scene—goals, methods, and accomplishments, W. Wright Memorial Lecture, Royal Aeronautical Society, London (April 28, 1949)].

E. D. Kane, USA

Thermodynamics

(See also Revs. 1732, 1768, 1780)

1776. **Ernst Schmidt, Thermodynamics: Principles and applications to engineering,** translated from 3rd German ed. by J. Kestin. Oxford University Press, 1949, xx + 532 pp. + 3 charts. Cloth, 5.5 × 8 in., $7.

The first German edition of this book appeared in 1936. This English edition is not a mere translation of the German third edition. Many changes and additions have been made to adapt it to English and American usage. A chapter on jet and rocket propulsion has been added, and the discussion of compressible fluid flow, and of gas and steam turbines has been extended to include up-to-date methods. The most recent data available have been used in the numerical tables and in calculations. British units have been introduced in the translation.

From the prefaces

1777. **O. Lutz, Enthalpies, entropies and equilibrium constants of gases of combustion** (in German), Ingen.-Arch. 16, no. 5–6 (1948).

The recent American compilations of thermodynamic properties of combustion gases are reviewed, and the tables are converted to units in general use in continental Europe. A bibliography on the subject is appended (124 references).

Serge Gratch, USA

1778. **Gordon E. Osterstrom, Knocking combustion observed in a spark-ignition engine with simultaneous direct and schlieren high-speed motion pictures and pressure records,** Nat. adv. Comm. Aero. Rep. no. 897, 19 pp. (1948).

Nine explosions were observed in a knocking experimental engine to determine the nature of events prior to the pressure pulses accompanying the knock. It was found that, under knocking conditions, when the flame front has progressed part way across the combustion space, the end gas appears to develop a more or less homogeneous reaction, which becomes increasingly intense up to the onset of the pressure pulsations, or knock. In one case this end gas reaction was observed when there was not a true knock. The direct photos showed luminous flame in the end gas several frames (40,000 frames per sec were used) before the knock, but the schlieren photos showed apparent reaction in the end gas region as much as 25 frames before the first evidence of luminosity. Early stages of autoignition were thus probably low-energy reactions even though quite extensive within the end zone.

The experimental engine, the direct and schlieren kinematographic technique, and the pressure recording system are described in detail.

A complicating factor arises from the fact that the fuel must be injected directly into the combustion chamber just before the explosion. Thus, there are local variations of fuel-to-air ratio which are not known. Use of a premixed fuel-air gas mixture entering the inlet port of the cylinder is not feasible because the window in the cylinder head then becomes coated with soot and may even crack from temperature stresses. Stewart Way, USA

1779. **Milo M. Bolstad and Richard C. Jordan, Theory and use of the capillary tube expansion device,** Refrig. Engng. 56, 519–523, 552 (Dec. 1948); 57, 577–583 (June 1949).

This paper contains useful data for the calculation of capillary-tube expansion devices which are frequently used for domestic-size refrigeration units.

In the first part the adiabatic expansion through a capillary tube is studied, and the results of a number of tests are listed. These results, as well as the theoretical approach, show that there is a close analogy between the phenomenon studied and the flow of a compressible gas through a pipe. Below a certain back pressure at the outlet of the device, the flow rate becomes independent of this back pressure, which can be left aside as a determining factor for most practical applications. Therefore the flow rate is only dependent on the inlet pressure and the inlet temperature.

The general case of nonadiabatic expansion is treated in the second part. The test setup was changed in order to allow for a cooling of the expanding refrigerant with the cold suction gases returning to the compressor. The results show that the flow rate is determined by the inlet pressure and the bubble point temperature (temperature corresponding to the point where the liquid reaches saturation). The charts presented allow a selection of the capillary tube which has to be used for a certain job. The accuracy of the charts is discussed especially with regard to the range of variation due to the oil content of the refrigerant.

E. Haenni, Switzerland

Heat and Mass Transfer

(See also Revs. 1618, 1756, 1779, 1805)

1780. **C. H. Coogan, Jr., Heat-transfer rates,** Mech. Engng. 71, 495–498 (June 1949).

The rate of heat transfer from the earth to a refrigerant flowing in copper tubes buried at various depths in the earth was measured over long periods of time. The data obtained are useful in the design of underground pipes of heat pumps and of other pieces of equipment which use the earth as a source of heat, or which are used primarily to cool sections of the earth.

It was found that a moist soil yielded higher rates of heat transfer to a buried tube than a dry soil, and that small tubes are

preferable to large tubes with the optimum size having a diameter between $^1/_2$ and $^3/_4$ in.

The measured temperature distribution in the soil around the buried pipes agreed reasonably well with the calculated values based on line-sink theory. Joseph Kaye, USA

1781. **Donald M. Vestal, Jr., Heat pump buried coil design—the soil problem,** Refrig. Engng. 57, 612–613 (June 1949).

The author discusses in general terms the effect of moisture migration on the heat transfer to cylindrical heat sources and sinks. A limited amount of field-test results are presented on the moisture and temperature gradients around a single evaporator coil buried at a depth of 4 ft. Some data on the rate of moisture migration in laboratory soil specimens are also given.

Y. S. Touloukian, USA

1782. **Harold S. Mickley and Charles A. Trilling, Heat transfer characteristics of fluidized beds,** Indust. Engng. Chem. 41, 1135–1147 (June 1949).

This paper reports the results of measurements on the rates of heat transfer radially either into, or out of, a vertical annular bed of small fluidized spheres. Heat-transfer coefficient is presented as a function of sphere diameter, rate of air flow, and solid concentration. The temperature within the bed away from the boundaries is quite constant.

The rate of heat transfer is much greater than would result from air flow alone, due apparently to the violently agitated motion of the solid spheres, but little is offered in the way of specific physical explanation. Stanley Corrsin, USA

Acoustics

1783. **R. S. Witte, B. A. Mrowca, and E. Guth, Propagation of audiofrequency sound in high polymers,** J. appl. Phys. 20, 481–485 (June 1949).

Velocity and attenuation measurements were made on butyl and GR-S gun stacks. The dynamic viscoelastic constants were obtained by setting up longitudinal waves through the sample, crystal transducers being used for transmitter and receiver. The velocity and attenuation were then determined by moving the pickup along the sample. The range covered frequencies from 0.5 to 5 Kc, and temperatures from -5 C to 90 C. Results show that the velocity increases with decreasing temperature and increasing frequency. The attenuation shows a peak with temperature. The behavior of the dynamic modulus is similar to the behavior of the velocity. Based on static measurements, the modulus is believed to increase at very low temperatures.

Robert Fehr, USA

1784. **Albert London, Transmission of reverberant sound through single walls,** J. Res. nat. Bur. Stands. 42, Rep. 1998, 605–615 (June 1949).

The author considers theoretically and experimentally the problem of the transmission of sound through a thin panel. The main results are based upon measurements of the transmission through $^1/_2$-in. plywood, and $^1/_2$, 1, and 2-in. plasterboard walls. It was concluded that the experimental results could be explained if it is assumed that the acoustic impedance of the wall is made up of three parts: (1) its mass (so-called "mass effect"), in which the wall vibrates as a whole, (2) its internal damping or dissipation, and (3) an effect due to the ability of the wall to propagate flexural vibration. The first and third points are well known, the latter being treated theoretically by Cremer [Akust. Z. 7, p. 81 (1942)]. The second point is a contribution of the author. The importance of random angles of incidence of the sound upon the wall are demonstrated both theoretically and experimentally. Finally, the author points out that there is little to be gained by covering the wall with paper straws having their axes normal to the wall, but a substantial gain is effected by gluing absorbent material such as hair felt to both sides of the wall.

Horace M. Trent, USA

1785. **A. W. Nolle, Electric-field modulation of ultrasonic signals in liquids,** J. appl. Phys. 20, 589–592 (June 1949).

The report deals with the effect of a periodic transverse electric field upon the properties of a continuous ultrasonic wave transmitted through a liquid. A 10-mc signal was applied to a quartz crystal in a small water tank. The resultant ultrasonic wave was transmitted through a 7-cm test cell containing the fluid under investigation and picked up by a second crystal, the output from which was fed into a narrow-band communications receiver. Two plane parallel electrodes 1.6 cm apart were attached to the sides of the test cell, and subjected to 400 cps alternating potential at voltages up to 1300 v. Either phase (frequency) or amplitude modulation of the ultrasonic wave could be detected with the receiver. Over twenty liquids, polar and nonpolar, conducting and nonconducting, were tested, with no evidence of amplitude modulation. Phase modulation was detected in all of the conducting (specific resistance $<5 \times 10^6$ ohm-cm) liquids and is attributed to a heating effect. None of the nonconducting (specific resistance $>10^8$ ohm-cm) liquids indicated phase modulation. The modulation observed in the conducting liquids is explained by the nature of the temperature changes that accompany the heating of a liquid by alternating current.

The author recommends the use of a longer path for the ultrasonic wave and somewhat lower frequencies, half-wave length cell walls to eliminate acoustic impedance mismatch, and operation to avoid unnecessary heating. Glenn Murphy, USA

1786. **D. K. C. MacDonald, Some statistical properties of random noise,** Proc. Camb. phil. Soc. 45, 368–372 (July 1949).

This paper is an attempt to analyze the phase correlation of random noise signals; that is, to determine the statistical properties of the phase difference ϕ between two signals separated in time by amount τ. Beginning with a distribution function obtained by Rice [Bell System tech. J. 24, p. 78 (1945)], the author obtains a distribution function for ϕ in a straightforward manner and applies it to the case of a Gaussian power spectrum. It is pointed out that a phase analysis on this basis is capable of supplying essentially more detailed information about the frequency spectrum than an amplitude analysis alone. Finally, the inversion from the phase or amplitude analysis to the power spectrum is considered. There appear to be some internal inconsistencies of notation which may prove troublesome to the reader.

Robert E. Roberson, USA

Ballistics, Detonics (Explosions)

(See also Revs. 1639, 1748, 1749)

1787. **Joachim Strecke, Mathematical interior ballistics** (in German), Naturforschung und Medizin in Deutschland 1939–1946, vol. 7, pp. 187–209. Dieterich'sche Verlagsbuchhandlung, Wiesbaden, 1948. DM10 ($2.40).

This is a review of progress by German ballisticians during the indicated period in the applied theory of interior ballistics. The earlier fundamental theory of Résal, Sarrau and Vieille provides adequate answers to the more immediate problems, and amply justifies predictions concerning maximum powder-gas pressures and muzzle energies. The influence of the viscosity of combustion

gas and of its eddying upon the temperature loss is not known. The theory must therefore as yet be content with idealized reactions and compute with linear gas flow. Recent investigations provided at least opportunity to formulate laws concerning the distribution of heat loss at the bore surface. The author reviews the dimensionless theory of Résal. Charbonnier's assumption is simplified for the modern powder with almost constant burning surface. The general linearized equations with certain further simplifying assumptions, such as assuming the covolume of the combustion gas to be equal to the specific volume of the fixed power, may be integrated in finite form to give the muzzle energy and maximum gas pressure. Theoretical results and observations not yet explained on theoretical grounds are discussed, particularly with regard to effect of the density of loading upon muzzle velocity. Theoretical and experimental results concerning the pressure difference upon the projectile base and the rear of the powder chamber due to flow of the gas and powder are given. Various theories as to the maximum possible muzzle velocity are discussed. The inertia of the air in the bore in front of the projectile has a noticeable effect.

Courtesy of Mathematical Reviews A. A. Bennett, USA

1788. J. Corner, The ballistic effects of bore resistance, Quart. J. Mech. appl. Math. 2, 232–245 (June 1949).

The resistance encountered by a projectile, as it moves down the bore of a gun, alters the pressure to which it is subject and the velocity it attains. This factor is usually neglected in ballistic theory. It is shown that the correction can be calculated for each point along the path of motion. The effect of bore resistance can be found rapidly for any given dependence of this resistance upon the coordinates of motion by integration of the product of resistance and a weighting factor which is related to the influence of resistance upon pressure. The method is not applicable if the resistance is sufficient to stop the projectile for a period of time, so that it cannot be employed near the start of motion. Frederick Seitz, USA

Soil Mechanics, Seepage

(See also Revs. 1633, 1705, 1735, 1736, 1800, 1804, 1805)

1789. H. Lorenz, Calculation of stress distribution under a rectangular loading surface (in German), Bauplan. Bautech. 3, 109–115 (Apr. 1949).

The author first reviews the fundamental theories of stress distribution and propagation in a solid mass under central loading. He points out the relation between Fröhlich's ν (factor of concentration) and the angle of stress extension φ_0, and states that the bigger values of φ_0 and lower values of ν are typical of cohesive soils, whereas granular soils may be characterized by higher ν and lower φ_0 values. Steinbrenner has solved the problem of stress distribution under uniformly loaded rectangular areas for $\nu = 3$, while the author solves the problem for higher ν values giving an exact and an approximate solution, presented also by means of diagrams. He points out that under similar conditions a higher stress will be set up in a subsoil having a higher ν value.

In the second part of his article, the author extends his formulas to eccentric loads taking up a linear distribution of pressure from 0 to σ_r along the width of the loading area. He derives formulas and diagrams for various positions of eccentric loads solving the problem according to the Steinbrenner principle, i.e., giving the stresses along the perpendicular of one corner of the rectangular area.

It is assumed, however, that the stress will attain its maximum value under one edge of the loading area without regard to its bearing capacity ("critical edge load" of Fröhlich).

Numerical examples illustrate the importance of the consideration of eccentricity in settlement analysis.

Ch. Széchy, Hungary

1790. A. L. L. Baker, Distribution of load on a group of piles, Informes Constr. 2, no. 13, article 483-5, 6 pp. (Aug.-Sept. 1949).

This is a Spanish translation of a paper published in the November 1948 issue of Concrete and Constructional Engineering. The author analyzes the complex loads acting on piles of any direction and gives formulas for the pile design. An example is included. Ed.

1791. Takeo Mogami, Determination of the bearing power of clay foundation, Proc. sec. int. Conf. Soil Mech. Found. Engng. 1, 55–57 (1948).

On the basis of certain assumptions and simplifications, the author derives a formula which he shows to be similar to that of Housel for load-bearing tests on clays. It is unfortunate that the somewhat rigorous analysis precludes consolidation and requires that the vertical stress under the bearing plate be perfectly uniform, as this renders the result no more than approximative. The soil being considered as a viscoelastic material, the settlement under load is made a function of a modulus of elasticity, coupled with the stress, and a modulus of viscosity, coupled with the stress and with time. This reviewer believes that since the author deals with strain velocity, the effect of consolidation may bear more heavily on the problem than he indicates.

Robert Quintal, Canada

1792. R. Haefeli and G. Amberg, Contribution to the theory of shrinking, Proc. sec. int. Conf. Soil Mech. Found. Engng. 1, 13–17 (June 1948).

This paper contains a theoretical analysis of the soil-shrinkage process. Mathematical expressions are formulated to describe this process. Theoretical results are shown to be in good agreement with the experimental ones. R. E. Fadum, USA

1793. W. J. Turnbull, S. J. Johnson, and A. A. Maxwell, Factors influencing compaction of soils, Highw. Res. Bd. Bull. no. 23, 1–11 (1949).

Of the variables affecting the compaction of soils, moisture content has been widely studied. The authors discuss the influence of other variables, e.g., the type of equipment used and the bearing capacity of the soil. As types of equipment, sheep's-foot rollers and rubber-tired rollers are studied.

For sheep's-foot rollers, the influence of many factors is discussed, such as: weight of roller, contact pressure, length, shape and spacing of feet, distribution of weight over different rows of feet, thickness of lift, and number of passes. An analysis of the motion of the feet in rolling is further given.

The conclusion is, that the heaviest roller available gives the best results, when the pressure under the feet does not exceed the bearing capacity of the soil. Therefore the construction of sheep's-foot rollers should include the possibility of adjusting the contact pressure by varying the contact area. The weight of the roller should only be decreased, when the contact pressure is still too high, with maximum contact area.

For rubber-tired rollers, the influence of contact pressure, thickness of lift, number of passes, and the distribution of the tire load on the soil are discussed.

The conclusion in this case is also, that the heaviest roller and the largest tire size available should be taken, but the bearing

capacity of the soil should not be exceeded. Therefore rollers should be constructed in such a way that variation of the contact pressure is possible. F. C. de Nie, Holland

1794. **Thos. E. Stanton, Vertical sand drains as a means of foundation consolidation and accelerating settlement of embankments over marsh land,** Proc. Sec. int. Conf. Soil Mech. Found. Engng. 5, 273–279 (June 1948).

The author presents a review of twenty-two achievements of this method which was already discussed at the first meeting in 1936. He shows the possibility of relieving hydraulic pressures, and greatly reducing the normal settlement period of embankments over deep mud deposits, by construction of 18- to 24-in-diam sand-filled vertical drains spaced approximately 10 ft on center, with resultant less inconvenience to traffic and lower maintenance cost.

The paper contains a description of the design and construction procedure and some of the problems encountered in connection with the method. Extensive preliminary investigation, proper design, and rigid enforcement of the specification are essential to success without delays in construction operations.

R. G. A. Spronck, Belgium

1795. **A. Rufenacht, Pore pressure assumptions for stability studies of earth dams,** Proc. sec. int. Conf. Soil Mech. Found. Engng. 3, 230–233 (June 1948).

The author proposes to approach the pore pressure in earth dams, during and after construction, by means of the consolidation theory of Terzaghi as applied to the process of sedimentation by Fröhlich, instead of assuming it to be a function of the overburden.

The assumptions to be made in this case are that this theory applies also to unsaturated material, on a nonporous base, and that drainage occurs in a vertically upward direction only. According to the author, because of these broad assumptions conservative values are obtained. F. C. de Nie, Holland

1796. **Richard Jelinek, The soil as a lateral-isotropic medium** (in German), Abh. Bodenmech. Grundbau, 19–23; Berlin, Erich Schmidt Verlag, 1948.

Although soils are generally assumed to be elastically isotropic, stratification or lamination introduces anisotropy in vertical planes while horizontal planes are frequently considered to be isotropic. For this special case of axisymmetric anisotropy the author has derived stress and strain equations in terms of the angle of inclination with the vertical axis of symmetry.

The results which have been obtained from the generalized Hooke's law and the relation between the elastic constants given by Green's energy function of distortion, are illustrated by polar diagrams showing the relation of the modulus of elasticity for various assumed relations of the modulus of rigidity. Unfortunately experimental methods do not exist at present to enable a check of these assumptions to be made.

G. G. Meyerhof, England

1797. **Richard Jelinek, The force propagation in the generalized plane state of stress for lateral-isotropic soils** (in German), Abh. Bodenmech. Grundbau, 24–27; Berlin, Erich Schmidt Verlag, 1948.

Under the assumptions of the preceding paper, the author derives general equations for the stresses due to a vertical concentrated live load on the surface of a semi-infinite solid. The results and the underlying Airy stress function, except in the special case of lateral incompressibility, are more rigorous than those obtained by Wolf [Z. angew. Math. Mech. 15, 249–254 (1935)], since equilibrium of the radial stresses with the load has been assured in the present derivation.

The distribution of the vertical stress on horizontal planes is shown for some typical cases and various assumed relations of the modulus of rigidity. The author's opinion that only these stresses are of interest in soil mechanics is not shared by the reviewer, who considers that the principal shearing stresses are equally important since they give a guide to the onset of plastic flow. G. G. Meyerhof, England

1798. **D. P. Krynine, Analysis of the latest American tests on soil capillarity,** Proc. sec. int. Conf. Soil Mech. Found. Engng. 3, 100–104 (1948).

Six series of experiments in capillarity conducted independently by six organizations are analyzed and summarized. The principal conclusions are: capillarimeters as applied to the soils tested were not satisfactory; the factor preceding the "rise function" (dependence of the time of capillary rise on the height) was found not constant for a given soil, probably because a function containing particle size is not included. These experiments were all conducted in the laboratory, and no mention is made of applicability of results to conditions encountered in the field.

Raphael C. Kazman, USA

1799. **R. Malcor, Flow with free surface in porous mediums,** Proc. sec. int. Conf. Soil Mech. Found. Engng. 7, 169–175 (June 1948).

The conformal transformation due to Helmholtz takes the free surface into a straight line in the hodograph plane. The problem of flow toward a horizontal drain reduces to a Dirichlet problem. Details and numerical computations are promised for later publication. This rigorous solution, like Dupuy's approximate one, yields a surface parabolic at infinity, which makes it useful within a certain distance only. Jean Goguel, France

Micromeritics

(See also Revs. 1734, 1735, 1754, 1755, 1782)

1800. **P. E. Raes and E. A. Loones, Granulometric analysis of soils by sedimentation on a floating cylinder** (in French), Géotechnique Lond. 1, 177–188 (June 1949).

A modification of the hydrometer method for the determination of the distribution of particle sizes of a finely divided material dispersed in a liquid is described. The float is a glass cylinder 38 cm long and 2 cm diam. The upper end is attached by a lever system to an indicator which magnifies the small vertical displacements during sedimentation. A similarly equipped smaller float which rests on the surface of the suspension is used to correct for changes in level due to sinking of the float and evaporation. The analysis is carried out by noting the levels of the floats at time intervals of 1, 2, 4, 8, and 16 minutes. These data together with density and viscosity values are sufficient to calculate the percentage of particles below any given diameter in a polydisperse suspension. The data and calculations are presented for a material with particles ranging in size from 7.7 to 86.7 microns.

C. Ross Bloomquist, USA

1801. **D. H. Belden and Louis S. Kassel, Pressure drops encountered in conveying particles of large diameter in vertical transfer lines,** Indust. Engng. Chem. 41, 1174–1178 (June 1949).

Experimental equipment is described, and data and analysis thereof with charts are presented for the pressure losses in vertical tubes through which granular particles are transported upward by gas. Particles used were oil-cracking catalysts of two size

ranges: 0.0274 to 0.0518, and 0.0566 to 0.0983 in. in diam. The small catalyst had a bulk density of 30.7 and a particle density of 53.7, while for the large one these values were 37.1 and 60.9 lb per cu ft, respectively.

Total pressure drop was expressed as the sum of static loss (i.e., loss due to drag on the particles required to overcome the weight of the particles in the fluid), and friction loss which is the additional loss when material is moved through the tube. Friction factors computed for the latter or friction part of the loss did not vary excessively, about 0.011 to 0.015. The bulk of data was well represented by the equation:

$$f\,(Re)^{0.2} = 0.049 + 0.22\,G_g\,G_c/(G_g + G_c)^2.$$

Friction loss is given by: $dP_f/dL = 2f\,(G_g + G_c)\,V_g/gD$, where f is friction factor, Re Reynolds number, G_g and G_c are mass velocities of the gas and catalyst in lb per sq ft per sec, P_f friction pressure drop lb per sq ft, L length in feet, D the tube diameter feet, and V_g the true gas velocity in fps. John C. Geyer, USA

1802. Leonard Farber, Flow characteristics of solids-gas mixtures, Indust. Engng. Chem. 41, 1184–1191 (June 1949).

This paper reports the results of experiments on the transportation of granular alumina-silica catalyst by flowing air in horizontal and vertical glass tubes 17-mm ID. The particle size of the catalyst, which had a specific gravity of 2.45, ranged from less than 10 microns to 200 microns. Measurements were made with four air-flow rates ranging from 0.0077 to 0.0152 lb per sec, with solids flow rates varying up to about 0.14 lb per sec. The principal observations were pressure drops which were measured in 2-ft long sections of the tubes for the various flow rates of air and solids.

In the vertical tube with a given air-flow rate the pressure drop increased continuously with the rate of flow of solids, while in the horizontal tube the pressure drop appeared to increase rapidly up to a certain value of the solid flow rate, and then to level off to a constant value. These latter measurements differ from results of studies by O'Brien and Folsom on transportation of sand by water flowing in a horizontal pipe, which show that, for a given flow velocity, the pressure drop measured in terms of feet of the fluid-solid mixture was constant and independent of the concentration of the solids. Vito A. Vanoni, USA

1803. W. K. Lewis, E. R. Gilliland, and W. C. Bauer, Characteristics of fluidized particles, Indust. Engng. Chem. 41, 1104–1117 (June 1949).

The batch and continuous fluidization of six different fractions of glass spheres, each with nearly constant diameter, have been investigated. The flow characteristics of the smallest size glass spheres (0.0016 in.) do not agree with those of the five larger sizes (0.0040 to 0.0224 in.). All these larger sizes behave similarly, and the data have been correlated by several methods.

C. Ross Bloomquist, USA

1804. Rollin D. Morse, Fluidization of granular solids, Indust. Engng. Chem. 41, 1117–1123 (June 1949).

Previously published data on fluid flow in fluidized-solid beds have been analyzed in an attempt to improve the understanding of the mechanism of fluidization. The Carman-Kozeny formula for pressure drop in fixed beds containing granular solids was used as the frame of reference for analyzing the behavior of fluidized beds. Observed differences between fluid flow in fixed and fluidized beds were explained in terms of flocculation of small particles, kinetic energy losses from turbulent motion of large particles, and the inherent instability of gas-fluidized systems. The need for uniformity of dispersion of fluid and particles in fluidized-solid chemical reactor performance was hypothesized and discussed. Richard H. Wilhelm, USA

1805. E. R. Gilliland and E. A. Mason, Gas and solid mixing in fluidized beds, Indust. Engng. Chem. 41, 1191–1196 (June 1949).

Fluidized beds in which finely divided solids are maintained in a continuously agitated condition by means of an upward stream of a gas are important in chemical reactors. The paper presents a qualitative approach to the problem of mixing of the gas and of the solid particles in such beds. Tubes of large length-to-diameter ratio were used experimentally.

Mixing in the gas phase was measured through the distribution of helium injected into the main gas stream. It was found that the amount of gaseous back mixing was small. A rough estimate was made of the turbulent eddy coefficient through helium concentration measurements at points upstream from the point of injection.

The extent of mixing of divided solids, using glass beads and catalyst microspheres, was estimated by interpretation of a heat-transfer experiment. A jacketed-tube heat exchanger was divided into two parts; the upper section was heated electrically and the lower was water cooled. Thermocouple explorations were made within the bed, and tube-wall heat-transfer coefficients for each section were computed in the presence and absence of particles. The results were interpreted in favor of a rapid mixing of the solid phase, tending to maintain a relatively constant temperature in that phase. Richard H. Wilhelm, USA

1806. H. P. Meissner and H. S. Mickley, Removal of mists and dusts from air by beds of fluidized solids, Indust. Engng. Chem. 41, 1238–1242 (June 1949).

Sulphuric-acid mist can be removed from air by passage through a bed of fluidized particles. Porous materials, such as microspheres, which are cracking catalysts, silica gel, and activated alumina, picked up 5% by weight of acid before fluidization was destroyed. The efficiency of acid removal was up to 85 to 95% at high gas velocities and remained constant during the life of the bed. The efficiency was independent of acid concentration over a wide initial concentration. Silica sand and glass beads showed a life of only a few minutes, since these materials are impervious and cannot absorb the sulphuric acids in their pores. The removal of ammonium-nitrate dusts can be effected by fluidized beds in a similar manner, but the preliminary data presented are not sufficient for generalization.

C. Ross Bloomquist, USA

Geophysics, Meteorology, Oceanography

1807. C. G. Rossby, On a mechanism for the release of potential energy in the atmosphere, J. Met. 6, 163–180 (June 1949).

The vorticity equations applied previously [Rev. 2, 1211] provide a means of calculating changes in vorticity of vertical air columns when displaced from one latitude to another, but they cannot give information about rates of change and durations of the displacements. In this paper a second set of equations is derived which gives the acceleration northward or southward, experienced by somewhat idealized individual vertical vortexes in the earth's atmosphere. Taken together these two sets of equations provide a much fuller analysis of the dynamics of atmospheric shells than if only the vorticity equations are applied.

It is shown that cyclonic vortexes over the pole are stable and, if displaced, perform stable nutations with a period of about one week, but that anticyclonic vortexes over the pole are unstable.

unless surrounded by a strong outer belt of westerly winds, and tend to move away from the pole with increasing speed. Thus a force exists which drives the cold polar anticyclones toward lower latitudes where their potential energy can be released and the cold air spread out in the lower layers.

The southward travel of anticyclonic cold domes in the lower troposphere gives rise to forced displacements southward of associated cyclonic vortexes in the upper troposphere and at the tropopause level. Finally, these composite vortexes reach an equilibrium latitude, and then they may even return slowly toward higher latitudes.

There are striking similarities between the streamline patterns at the anticyclonic base of such sinking cold domes, as calculated by this theory, and as shown in synoptic charts for three consecutive days. Wm. C. Johnson, Jr. USA

1808. L. N. Gutman, On the theory of mountain-valley winds (in Russian), Doklady Akad. Nauk SSSR 66, no. 2, 199–202 (May 1949).

Considering a special type of mountain in which the slope is constant along an arbitrary direction, the author uses a system of orthogonal curvilinear coordinates, where the x-axis is always directed along the slope. In such a system all elements then depend primarily upon the x-coordinate. To eliminate the effect of thermal turbulence, he assumes further that the temperature on the surface of the mountain is everywhere the same, and that the slope is sufficient for applying the theory of boundary layer. Under these assumptions the author obtains for the components of velocity and the deviation of the potential temperature from its undisturbed value (without any motion) a system of equations, derived from the equation for a compressible viscous fluid, applying the theory of boundary layer and the theory of convection, following the method of A. Landau and E. Lifshitz (*Mekhanika sploshnykh sred*, 1944). This system is then simplified by omitting small terms and introducing other restrictive assumptions (coefficient of viscosity and thermal conductivity constant, quasi-stationary motion, etc.). By means of some characteristic quantities the system is transformed to a nondimensional form with only numerical coefficients, which the author tries to solve by the method of separation of variables, expanding the solution in series in x, the coefficients in which are then given by a system of nonlinear ordinary differential equations. These equations can be solved successively and the author gives the solution for the first coefficients, representing the first approximation for the vertical velocity and the deviation of potential temperature. The vertical distribution of these quantities is plotted. No other conclusion about the characteristics of mountain-valley winds is given.

Z. Sekera, USA

©1809. **Daniel W. Mead, Hydrology: The fundamental basis of hydraulic engineering,** 2nd ed., New York and London, McGraw-Hill Book Co., Inc., 1950, xviii + 728 pp. Cloth, 5.5 x 7.7 in., \$6.

Minor corrections, revisions, and additions have been made in virtually every section of the first edition, and in many instances entire sections have been added or completely rewritten. The Bergeron analysis of meteorological phenomena in terms of air masses has been discussed in chapter IV. The more recent approach to evaporation theory and methods of measurement appear in chapter VI. The new techniques and possible consequences of producing artificial rainfall are mentioned. The methods of weighting precipitation records for given areas and, in particular, the Thiessen method, have been added to chapter IX. The application of the theory of probability to hydrologic data is discussed in two new sections in chapters IX and XX. In chapter XII on drought, by Henry J. Hunt, the effects of rainfall characteristics of areas in this country and abroad on droughts and periods of excessive precipitation are analyzed. The ever-increasing demands on ground-water sources in the past few decades have been reappraised in Chapter XVI. Chapter XX has been enlarged by the discussion of the rational method of computing small area peak flows, and the unit-hydrograph and relating flood-routing procedures. From the preface

Lubrication; Bearings; Wear

(See also Revs. 1618, 1756, 1779, 1805)

1810. W. F. Cope, The hydrodynamical theory of film lubrication, Proc. roy. Soc. Lond. Ser. A 197, 201–217 (June 1949).

This paper is a re-examination of the theory of hydrodynamic lubrication between two surfaces in steady relative motion. Consideration of the relative magnitudes of the various terms in the equations for typical cases allows considerable simplification. Thus the inertia terms in the momentum equation, and the dilation and conductivity terms in the energy equations are neglected. It is noted that the exact solution of the equations requires an equation of state which is simple only for a gas. It is shown that, to obtain a suitable bearing-pressure distribution with a gas as lubricant, the geometric slope will have to be considerable to overcome the effect of density variation, while with a liquid the density variation with pressure is small compared to its decrease with temperature, and a parallel-face bearing is possible. Thus a gas requires a "geometric wedge" bearing in which the distance between the two surfaces decreases in the direction of motion, while a liquid will work in a parallel-face bearing due to a "thermal wedge" in which the density decreases in the direction of motion.

With the aid of simplifying assumptions (side leakage neglected, analytically simple equations of state and viscosity laws) several cases have been solved, and their results are summarized. The load-carrying capacities of the two wedges are shown to be about the same when the viscosity varies greatly with temperature, but the thermal wedge is shown to be superior if the viscosity variation is small. The thermal wedge is mechanically simpler, but the required clearances are small so that efficient fluid filtration is required.

The author concludes by noting that successful film lubrication requires a wedging action of some kind, either geometric or thermal. The first type is important in many applications, but the second may altogether outclass it if the conditions of small viscosity variation with temperature, large coefficient of cubical expansion, and small clearance are obtained.

J. M. Robertson, USA

1811. Charles D. Strang and Thomas P. Clark, Effect of high shear rate on erosion of common bearing metal, Nat. adv. Comm. Aero. tech. Note no. 1887, 25 pp. (June 1949).

A typical aircraft oil was forced under high pressure gradients through a narrow gap for periods of 6 hr. One of the gap faces was of steel, the other of copper, silver, or lead. The mean surface rates of shear on the solid surfaces reached values of 19×10^6 sec^{-1}, corresponding to a shear stress at the surface of 43 lb per sq in. With filtered oil the specimens of copper, silver, and lead showed no signs of erosion or other surface damage. Surface finish marks, scratches, and indentations failed to act as nuclei for erosion.

With unfiltered oil a "pseudoerosion" resembling that found in aircraft power plants was observed. This is due to the forcing of relatively large solid particles through the flow path. Another

type of erosion by particles smaller than the oil-filter thickness was also observed. This occurs by a "sand-blasting" action of the particles at regions where the oil flow changes its direction rapidly.

In addition it was found that under these conditions of flow the oil exhibited anomalously high rates of flow at the higher rates of shear.

D. Tabor, England

Marine Engineering Problems

(See also Revs. 1726, 1769)

1812. W. C. S. Wigley and J. K. Lunde, Calculated and observed wave resistances for a series of forms of fuller midsection, Trans. Instn. nav. Archit. Lond. 90, 92–110 (1948).

The calculation of wave resistance on the basis of the equations and methods derived by Havelock and Wigley from Michell's well-known integral for the flow potential, meets difficulties with forms having a full mid-section coefficient. The authors develop a method of calculation, corrected for the effect of viscosity on wave making, for these full forms, and find satisfactory agreement with the results of experiments carried out in the Teddington and Trondheim Ship Model Basins. Of course the flat bottom violates the basic assumptions of Michell's integral, but the authors believe that their method now overcomes this difficulty, and that it can produce results as favorable as hitherto obtained for fine forms. This is to be considered an important step forward, although the problem is still simplified by the assumption of geometrically similar cross sections of the forms and a symmetry about the midship section. The development of Wigley's life work on this subject is very promising indeed.

L. Troost, Holland

1813. J. F. Allan and W. P. Walker, Resistance of barges in deep and shallow water, Trans. Instn. nav. Archit. Lond. 90, 154–167 (1948).

With increasing construction of inland lakes and waterways, and the economic demands for low-cost tonnage transportation, this timely description of barge testing and performance should be a welcome contribution. The lines of four or five distinctive types of barges are given (one, similar to a ship) together with comparison curves characteristic of resistances at various "speed-length" ratios. The basic curves for single barges, in deep water, are accompanied by curves for pairs of barges towed abreast, and for tandem fleeting, and at various spacings. An important feature is the comparison set of curves for shoaling depths, including an extreme where depth equals 1.17 times draft.

From tank tests, resistance results are shown, as for 100-ft length, of effective horsepower, and the (British) coefficient encircled "C," plotted on a base of $V/L^{1/2}$. A separate diagram for shallow-water increments, on the various shapes and combinations, is shown for various D/d (depth/draft) ratios, at non-wave-making speeds.

In one group of tests, in two models each, in 12-ft lengths, of ship-shape, spoon-shape, swim-ended, and scow-type barges, there were single-barge curves showing spoon shape with the lowest ehp over the speed range, and scow shape the worst. Fleeting in tandem showed a reduction in resistance. Corresponding, average interference factors (IF) are given.

The speed range below a rapid increase in wave-making resistance was taken up to $V^2 = 3D$, where V is speed in knots, and D depth of water in feet. This gives a speed equal to half that of the wave of translation. Technique for tank depth was discussed.

In a second set of tests, models 7.7 ft long were used. In making comparisons between different sizes, it was found that Schoenherr's coefficients brought agreement of results within an average of 2%. R. E. Froude's coefficients departed by 10%. Accordingly, Schoenherr's coefficients were used to take account of model lengths, and then a 10% additional correction could refer the results to the Froude method, for ehp comparisons, for the normal-sized, larger models.

For the benefit of those not ordinarily using the encircled coefficient "C," we add that it is defined by the equation

$$(C) = 427.1 \text{ ehp}/(\text{displacement})^{2/3} V^3$$

where displacement is in tons, V is speed in knots; it also may be found from several other convenient relations.

Eastman Smith, USA

1814. J. L. Kent, The causes and prevention of slamming on ships, Engineering 167, 582–583 (June 1949).

Slamming of ships' hulls, or the violent impacts in rough seas, often followed by high-frequency vibrations of the entire ship, is of concern to the naval architect as a cause of damage to the ship's plating, and of deterioration of its structure due to fatigue.

An investigation of hulls of various forebody shapes towed in waves of different height, length, and shape, conducted in the tank of the National Physical Laboratory at Teddington combined with observations on vessels at sea, led to the following findings:

Conditions favorable to heaving and dipping produce slamming easier than those favorable to pitching. These occur as the wave length of the sea swell is approximately 1.73 times the length of the ship, and the period of encounter of the waves approximately equal to the period of dipping. Slamming is caused by sudden changes of the hydrodynamic pressure along the hull. Hulls with full lines, with a sharp turn at the bilge, with a flat bottom are most susceptible, likewise ships traveling at light draft. In order to reduce slamming, transverse sections of the foremost $^1/_8$ length of the ship, the portion known to be subject to damage, should be designed with a sharp rise of the floor and a large radius of the bilge. Reduction of the speed of the ship will always reduce or eliminate slamming.

J. R. Weske, USA

1815. Jean Lefol, Interactions between hull and propeller (in French), Bull. Ass. tech. Marit. Aéro. 46, 221–251 (1947).

This is a study of the elements of wake in the propeller disk, and of the increase of ship resistance caused by the working propeller (thrust deduction). The total propulsive efficiency is split up into the well-known Froude factors, which are discussed theoretically, with special regard to the relation between wake and thrust deduction. The frictional, potential, and orbital influences are taken into account, and it is concluded that there is practically no scale effect in the thrust-deduction factor, although there is no doubt that it is present in the wake coefficient.

Various methods of measuring wake at small-scale ship models are discussed as well as the existing empirical formulas. The author develops a new method for determining these factors and presents them in diagrams for basic dimensions to be corrected for any practical case. Numerical examples are given.

The author finds himself in disagreement with Horn's theoretical results, but the reviewer considers the author's deduction to be erroneous, although his diagrams may be useful for practical application.

L. Troost, Holland